Random focusing of tsunami waves

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Random focusing from ocean floor

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LETTERS

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Tsunamis exhibit surprisingly strong height fluctuations. An in-depth understanding of the mechanisms that lead to these variations in wave height is a prerequisite for reliable tsunami forecasting. It is known, for example, that the presence of large underwater islands[1] or the shape of the tsunami source[2] can affect the wave heights. Here we show that the consecutive effect of even tiny fluctuations in the profile of the ocean floor (the bathymetry) can cause unexpectedly strong fluctuations in the wave height of tsunamis, with maxima several times higher than the average wave height. A novel approach combining stochastic caustic theory and shallow water wave dynamics allows us to determine the typical propagation distance at which the strongly focused waves appear. We demonstrate that owing to this mechanism the small errors present in bathymetry measurements can lead to drastic variations in predicted tsunami heights. Our results show that a precise knowledge of the ocean's bathymetry is absolutely indispensable for reliable tsunami forecasts.

Introduction

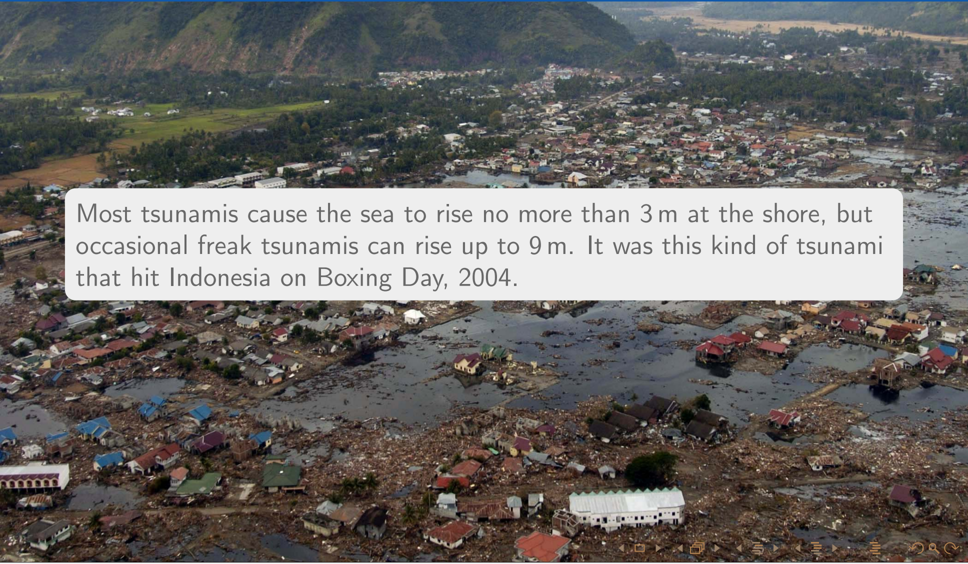
Ocean waves

- Caused by wind, tides
- Wavelengths up to a few 100 m
- Heights up to 10s of metres
- Travel up to 80 $\mathrm{km\,h^{-1}}$

Tsunamis

- Caused by geological activity
- Wavelengths typically 10s of km
- Amplitudes in open ocean typically < 1 m (only become big in shallow water)
- Travel up to 800 $\mathrm{km\,h^{-1}}$ (as fast as an A380)

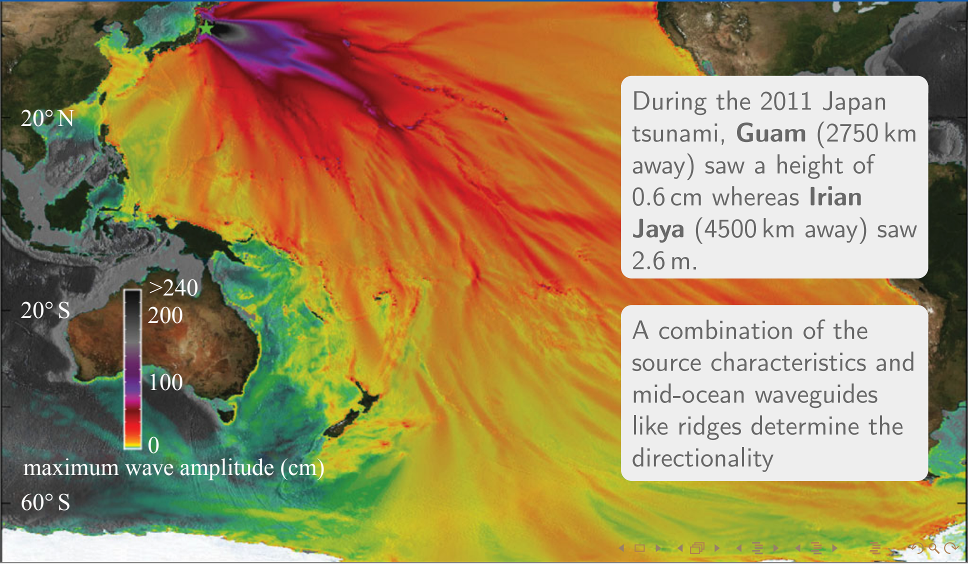
The largest tsunamis



Most tsunamis cause the sea to rise no more than 3 m at the shore, but occasional freak tsunamis can rise up to 9 m. It was this kind of tsunami that hit Indonesia on Boxing Day, 2004.

Animation

Difficult to predict



During the 2011 Japan tsunami, **Guam** (2750 km away) saw a height of 0.6 cm whereas **Irian Jaya** (4500 km away) saw 2.6 m.

A combination of the source characteristics and mid-ocean waveguides like ridges determine the directionality

How do we model its propagation?

Modelling wave propagation

Let's define the amplitude of a wave in time and space to be:

$$\eta\left(\vec{x}, t\right) \tag{1}$$

In optics, we model its propagation with the wave equation:

$$\underbrace{\frac{\delta^2 \eta\left(\vec{x}, t\right)}{\delta t^2}}_{\text{time dependent part}} = c^2 \underbrace{\nabla^2 \eta\left(\vec{x}, t\right)}_{\text{space dependent part}} \tag{2}$$

where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (speed of light)

Modelling wave propagation

In water, the same equation applies, but with a different constant c (the *shallow water equation*):

$$\frac{\delta^2 \eta\left(\vec{x}, t\right)}{\delta t^2} = c^2 \nabla^2 \eta\left(\vec{x}, t\right) \tag{3}$$

where $c = \sqrt{gH_0}$, linking with gravity g and average ocean depth H_0. This is the phase velocity: if $H_0 \approx 4\,\mathrm{km}$ then $c \approx 720\,\mathrm{km\,h^{-1}}$.

In this case $\eta\left(\vec{x}, t\right)$ describes the surface elevation normalised by the average depth.

Rays

Since c is time independent, the solutions are linear combinations of waves of the form:

$$\eta\left(\vec{x}, t\right) = A_1\left(\vec{x}, \omega\right)\cos\left(\omega t\right) + A_2\left(\vec{x}, \omega\right)\sin\left(\omega t\right), \tag{4}$$

and we can rewrite this as:

$$\eta\left(\vec{x}, t\right) = A\left(\vec{x}\right)\cos\left(k\vec{x} - \omega t\right), \tag{5}$$

where A is the amplitude part and $k\vec{x} - \omega t$ is the phase part.

Rays

$$\eta\left(\vec{x}, t\right) = A\left(\vec{x}\right) \cos\left(k\vec{x} - \omega t\right)$$

If we assume that c and ω vary slowly with $\vec{x}$, i.e.:

- Density of water reasonably stable
- No sub-wavelength structure

then the direction of propagation k is itself almost constant ($k = \frac{\omega}{c}$) and we can **borrow the concept of rays from geometrical optics**.

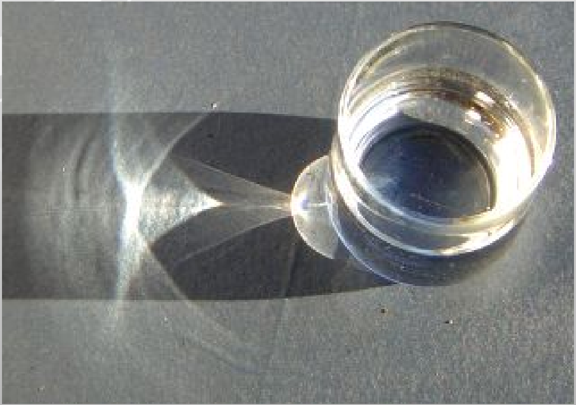
Rays

Rays can be focused!

The most destructive tsunamis occur when "rays" of the tsunami combine constructively, leading to wave heights of up to 9 m.

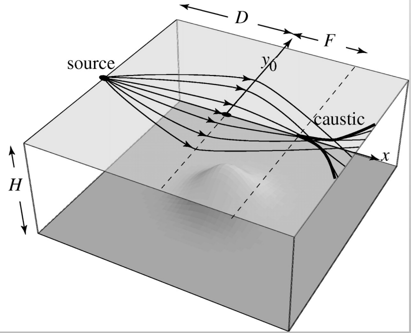
Caustics

When focusing occurs, it is typically into a "caustic" rather than a focal point - the same effect as spherical abberation on an optical lens.



Caustics

Caustics can occur when shallow regions contain large underwater structure such as mountains



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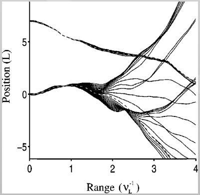
The authors of this paper argue caustics can appear due to small fluctuations in the height of the ocean floor

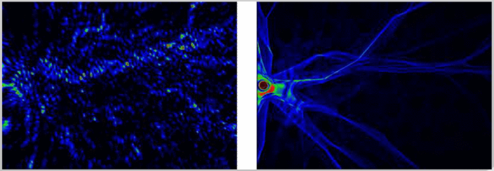
Random focusing from ocean floor

They relate the effect of **branched flow**:

Electron wave propagation through high-mobility semiconductors



Sound propagation through oceans



Microwave transmission through field of weak random scatterers

Modelling wave propagation

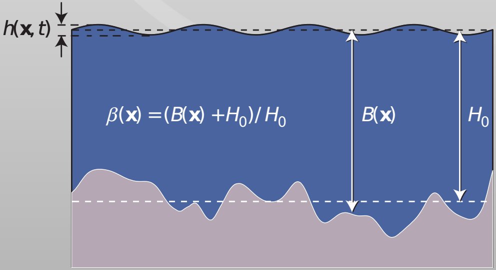
Effect of the ocean floor is well described with a small modification to the shallow water equation:

$$\frac{\delta^2 \eta\left(\vec{x}, t\right)}{\delta t^2} = c^2 \underbrace{\left(1 - \beta\left(\vec{x}\right)\right)}_{\text{new bit}} \nabla^2 \eta\left(\vec{x}, t\right) \tag{6}$$

Modelling wave propagation

$$\frac{\delta^2 \eta(\vec{x}, t)}{\delta t^2} = c^2 \underbrace{(1 - \beta(\vec{x}))}_{\text{new bit}} \nabla^2 \eta(\vec{x}, t)$$

Here $\beta(\vec{x})$ is the *fractional surface elevation of the ocean floor* or **bathymetry**.



The bathymetry $\beta(\vec{x})$ helps to govern the rate at which the surface height diverges in space and time.

Rays

$$c_R\left(\vec{x}_R\left(t\right)\right) = c\left(\vec{x}_R\left(t\right)\right)$$

The ray picture is valid as long as the wavelength is shorter than the correlation length of the bathymetry's random structure.

c.f. near field / far field

Rays

From the assumption of a geometrical limit (i.e. ray optics) comes the **ray equations**,

$$\dot{\vec{x}} = \left(1 - \beta\left(\vec{x}\right)\right)\vec{p} \tag{7}$$

$$\dot{\vec{p}} = \frac{c^2 \nabla \beta\left(\vec{x}\right)}{2\left(1 - \beta\left(\vec{x}\right)\right)}, \tag{8}$$

independent of the wave number k. In $\dot{\vec{x}}$ there is a "noise" term $\beta\left(\vec{x}\right)\vec{p}$ that represents the bathymetry.

Rays

Consider two rays that focus at some caustic. They each follow a path over a bathymetry which is some random noise that follows a correlation function (e.g. Gaussian). Then the ray equations can be rewritten as:

$$\vec{x} = (1 - \beta_0 \Gamma_1 (t)) \tag{9}$$

$$\vec{p} = \frac{c^2 \beta_0 \Gamma_2 (t)}{\sqrt{2} l_c} \tag{10}$$

where $\beta_0 \Gamma_1$ and $\beta_0 \Gamma_2$ are noise terms and l_c is the correlation length of the random noise function.

Rays

$$\vec{x} = \left(1 - \beta_0 \Gamma_1\left(t\right)\right)$$
$$\vec{p} = \frac{c^2 \beta_0 \Gamma_2\left(t\right)}{\sqrt{2} l_c}$$

This has no known solution, but the authors can instead calculate its moments (a combination of a physical quantity and a distance).

Rays

Skipping some maths, the main result is that the typical distance l_f to the first caustic is:

$$l_f = \alpha l_c \langle \beta^2 \rangle^{-\frac{1}{3}} \tag{11}$$

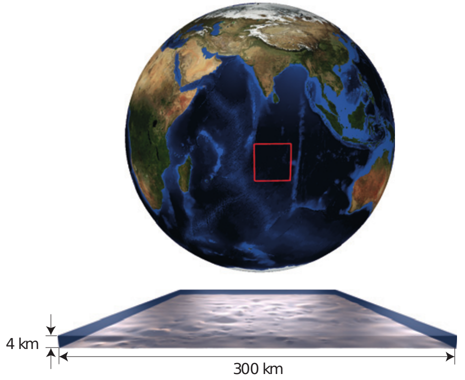
Here α is a scaling factor that depends on the shape of the source and $\langle \beta^2 \rangle$ is the average value of the square of the bathymetry ($\langle \beta^2 \rangle \ll 1$).

But does it work in practice?

Experimental verification

The authors applied tsunami propagation to a region of the Indian Ocean:

- 4 km deep
- Area free from islands and high underwater structures
- Bathymetry std. dev. 6.9 % of depth



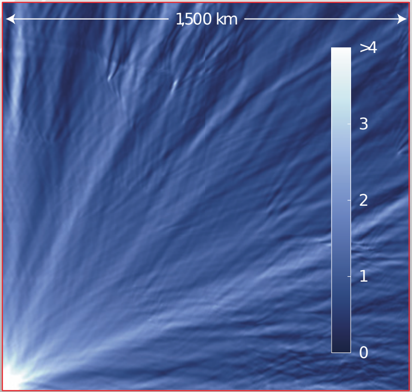
Video

"Because the [bathymetry] is correlated, neighbouring rays will initially travel in the same direction. Only when they have travelled far enough in the main propagation direction will rays start to intersect and focusing will occur."

Video

Effect of bathymetry on propagation

Even though $\sigma = 6.9\,\%$ of depth, surface heights of 6 times the average can be found in the simulation results, caused by random focusing.



Uncertainty

The ocean floor is typically mapped using echo sounding from ships or gravitational measurements from space.

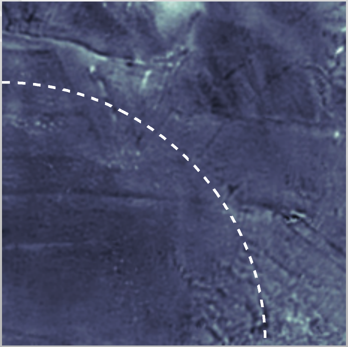
The uncertainty in these measurements is typically of the order of hundreds of metres.

What effect do these uncertainties have on the model?

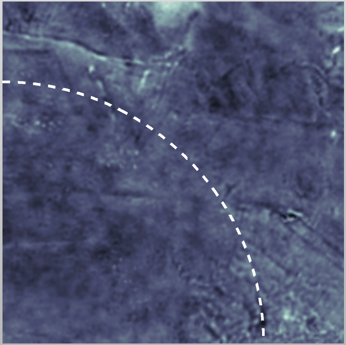
Uncertainty

The authors added fluctuations with variance 4% to the bathymetry:

before

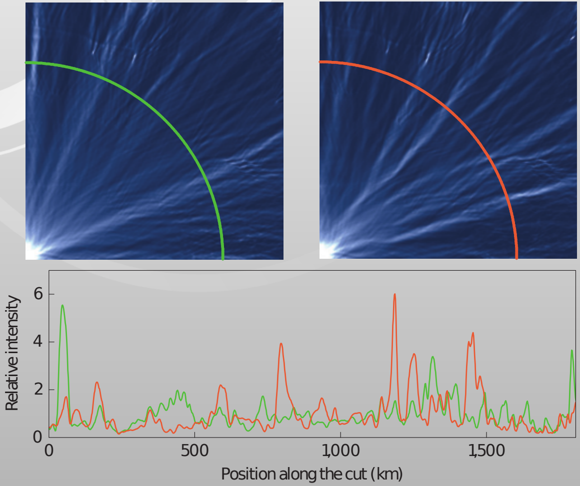


after

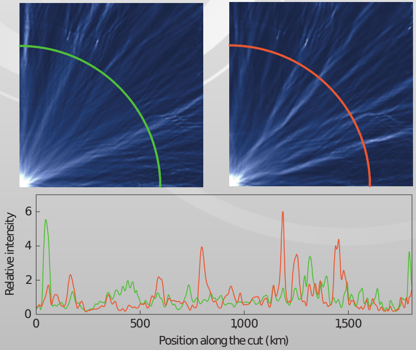


Uncertainty

And this dramatically changed the locations of the caustics:

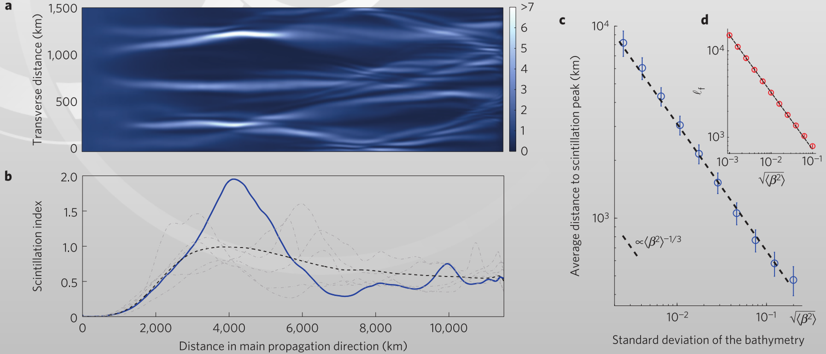


Uncertainty



This emphasises how important it is to have accurate bathymetry profiles to predict where caustics might occur.

Different wavelengths



Scintillation index is the ratio of std. dev. of signal to average signal.

Executive summary

- Tsunamis are destructive
- Their propagation is hard to predict
- But they can be treated like geometric optics
- Macroscopic structures focus tsunamis
- But ocean floor structure does so too
- A model can give you the typical distance to focus events
- But you need really good maps of the ocean floor

Questions?

(Please be nice!)

Extra slides

Rays

$$\eta\left(\vec{x}, t\right) = A\left(\vec{x}\right)\cos\left(L\left(\vec{x}\right) - \omega t\right)$$

New definition of L

Rays

We can now rearrange the propagation equation and use complex notation:

$$\left(\frac{\delta^2}{\delta t^2} - \nabla c^2 \nabla\right) e^{(\alpha + iL(\vec{x}) - i\omega t)} = 0 \tag{12}$$

where $\alpha = \ln A$

and we can write real and imaginary parts separately:

$$\omega^2 - c^2 \nabla L \nabla L + c^2 \nabla \alpha \nabla \alpha + \nabla c^2 \nabla \alpha = 0 \tag{13}$$

$$2c^2 \nabla \alpha \nabla L + \nabla c^2 \nabla L = 0 \tag{14}$$

Rays

$$\omega^2 - c^2 \nabla L \nabla L + c^2 \nabla \alpha \nabla \alpha + \nabla c^2 \nabla \alpha = 0 \tag{15}$$

$$2c^2 \nabla \alpha \nabla L + \nabla c^2 \nabla L = 0 \tag{16}$$

Note that $\nabla L = k$ where k is the wave number. Taking the limit where c and α vary slowly w.r.t. the wavelength,

$$\nabla L \nabla L = \frac{\omega^2}{c^2} \tag{17}$$