DEEP FREQUENCY MODULATION INTERFEROMETRY

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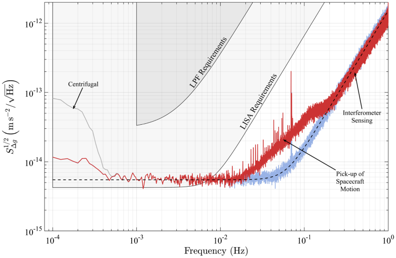
INTRODUCTION

LISA PATHFINDER'S INTERFEROMETER

- State of the art in low frequency strain measurement
- Complex arrangement of quasi-monolithic optics
- Only measures one degree of freedom; LISA will need to measure 12 per spacecraft
- Transferring mass into space is expensive
- Can we reduce the complexity?



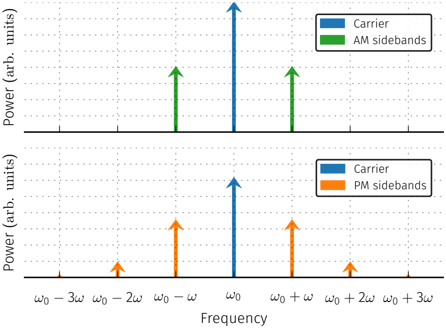
REDUCED COMPLEXITY PHASE MEASUREMENT

MODULATION

Sinusoidal signal encountering a sinusoidal phase modulation (e.g. an oscillating mirror) has power

$$P = P_0 \cos\left(\phi + \omega_0 t + m\cos\left(\omega_\mathrm{m} t + \psi_\mathrm{m}\right)\right) \tag{1}$$

where ϕ is an arbitrary phase offset and m is the *modulation depth* (or *index*).



Typically in our field, we use small modulation depths ($m \leq 0.3$) for RF locking, where only the first few sidebands are important

DIGITALLY ENHANCED INTERFEROMETRY

- Impose phase modulated pseudo-random noise onto the input light
- Signals for each mirror can be extracted in post-processing
- See Christian's journal club from 2015-10-30
- Performance levels reach 10 pm $\sqrt{\text{Hz}}^{-1}$ at low frequencies
- Small microchips don't offer this performance yet, but it's promising

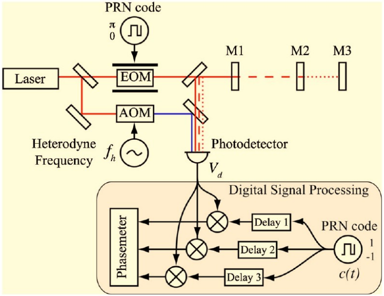
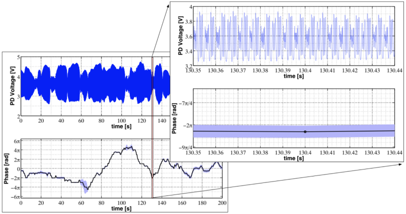


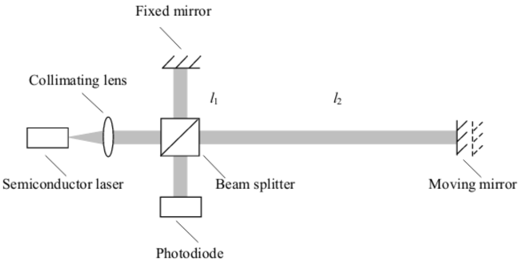
Fig. 1. (Color online) Digitally enhanced heterodyne interferometer for monitoring the separation of mirrors M1, M2, and M3. Reflections from the different mirrors are isolated by matching the decoding delays to the optical delays. EOM, electro-optic modulator; AOM, acousto-optic modulator; PRN, pseudorandom noise.

PHASE, not FREQUENCY

- Imposes strong sinusoidal phase modulation (i.e. m is not small) on light in one arm
- Leads to a **comb** of beats, not just a few as with weak phase modulation
- Non-linear fit algorithm extracts amplitude of beats to determine phase
- Demonstrated 20 pm $\sqrt{\text{Hz}}^{-1}$ sensitivity at mHz



- "Self homodyning" occurs when unequal arms allow the LO to be imposed on the laser at source instead of in one arm
- Techniques similar to digital interferometry, but using frequency modulation instead of phase modulation, achieved $1\,\mathrm{pm}\,\sqrt{\mathrm{Hz}}^{-1}$ above 1 Hz



DEEP FREQUENCY MODULATION

Deep frequency modulation interferometry

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Abstract: Laser interferometry with pm/$\sqrt{\text{Hz}}$ precision and multi-fringe dynamic range at low frequencies is a core technology to measure the motion of various objects (test masses) in space and ground based experiments for gravitational wave detection and geodesy. Even though available interferometer schemes are well understood, their construction remains complex, often involving, for example, the need to build quasi-monolithic optical benches with dozens of components. In recent years techniques have been investigated that aim to reduce this complexity by combining phase modulation techniques with sophisticated digital readout algorithms. This article presents a new scheme that uses strong laser frequency modulations in combination with the deep phase modulation readout algorithm to construct simpler and easily scalable interferometers.

Experimental demonstration of deep frequency modulation interferometry

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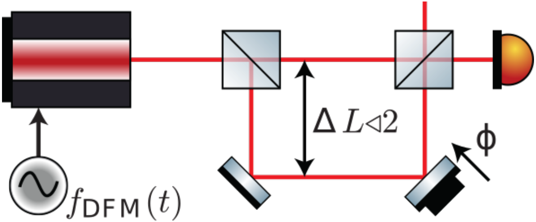
Abstract: Experiments for space and ground-based gravitational wave detectors often require a large dynamic range interferometric position readout of test masses with 1 pm/$\sqrt{\text{Hz}}$ precision over long time scales. Heterodyne interferometer schemes that achieve such precisions are available, but they require complex optical set-ups, limiting their scalability for multiple channels. This article presents the first experimental results on deep frequency modulation interferometry, a new technique that combines sinusoidal laser frequency modulation in unequal arm length interferometers with a non-linear fit algorithm. We have tested the technique in a Michelson and a Mach-Zehnder Interferometer topology, respectively, demonstrated continuous phase tracking of a moving mirror and achieved a performance equivalent to a displacement sensitivity of 250 pm/$\sqrt{\text{Hz}}$ at 1 mHz between the phase measurements of two photodetectors monitoring the same optical signal. By performing time series fitting of the extracted interference signals, we measured that the linearity of the laser frequency modulation is on the order of 2% for the laser source used.

In a nutshell, idea is to combine self-homodyning frequency modulation with deep phase modulation's readout (i.e. the non-linear fit algorithm)



- Unequal arms separated by ΔL
- The laser's frequency is intentionally modulated:

$$f_{\text{DFM}} = \Delta f \cos\left(2\pi f_{\text{m}} t + \psi_{\text{m}}\right), \tag{2}$$

where Δf is the modulation depth, f_{m} is the frequency modulation and ψ_{m} is the modulation phase.

The detector sees two terms, one from each arm:

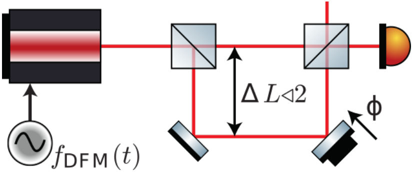
Short arm

$$E_{\mathrm{s}} = \frac{1}{2} E_{\mathrm{in}} \sin\left(\omega t + \frac{\Delta f}{f_{\mathrm{m}}} \sin\left(\omega_{\mathrm{m}} t + \psi_{\mathrm{m}}\right) + C\right) \tag{3}$$

Long arm

$$E_{\mathrm{l}} = \frac{1}{2} E_{\mathrm{in}} \sin\left(\omega\left(t - \tau\right) + \frac{\Delta f}{f_{\mathrm{m}}} \sin\left(\omega_{\mathrm{m}}\left(t - \tau\right) + \psi_{\mathrm{m}}\right) + C - \phi\right) \tag{4}$$

τ is the light travel time in the long arm, C is an arbitrary constant phase term, and ϕ is the gravitational wave signal.



The signal power at the detector is then $P_{\mathrm{out}} \propto (E_{\mathrm{s}} + E_{\mathrm{l}})^2$, i.e.

$$P_{\mathrm{out}} = \frac{P_{\mathrm{in}}}{2} + \frac{P_{\mathrm{in}}}{2} \cos\left(\omega_0 \tau + \phi + \frac{\Delta f}{f_{\mathrm{m}}} \left(\sin\left(\omega_{\mathrm{m}} t + \psi_{\mathrm{m}}\right) - \sin\left(\omega_{\mathrm{m}}\left(t - \tau\right) + \psi_{\mathrm{m}}\right)\right)\right) \tag{5}$$

Assuming the delay $\omega_0 \tau$ is small, this approximates to

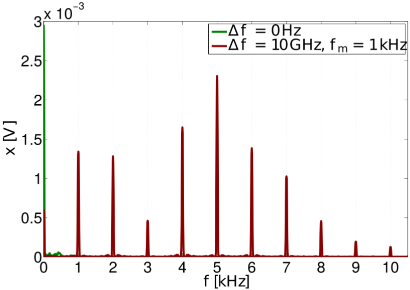
Output signal

$$P_{\mathrm{out}} = \frac{P_{\mathrm{in}}}{2} + \frac{P_{\mathrm{in}}}{2} \cos\left(\phi + 2\pi \Delta f \tau \cos\left(\omega_{\mathrm{m}} t + \psi_{\mathrm{m}}\right)\right) \tag{6}$$

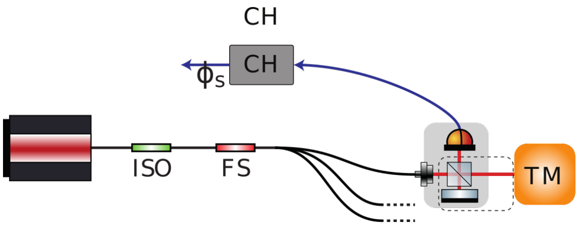
Output signal

$$P_{\text{out}} = \frac{P_{\text{in}}}{2} + \frac{P_{\text{in}}}{2} \cos\left(\phi + 2\pi\Delta f\tau \cos\left(\omega_{\text{m}} t + \psi_{\text{m}}\right)\right) \tag{7}$$

This is the same as phase modulation discussed earlier (neglecting $\omega_0 t$), but with $m = 2\pi\Delta f\tau$, **i.e. the amount of phase modulation scales linearly with the arm length difference and the frequency modulation depth**.



Simple optical setup:



- The laser light is split into fibres going to various “optical head” interferometers
- A large number of heads can share the same laser
- Photodetectors can be standard or quadrant depending on purpose
- Signal extraction via transimpedance amplifiers

Signals are demodulated with sine and cosine signals at N harmonics of ω_{m}, then low-pass filtered:

$$\begin{aligned} Q_{\mathrm{n}} &= v_{\mathrm{out}}(t)\cos(n\omega_{\mathrm{m}}t) \\ &\approx kJ_{\mathrm{n}}(m)\cos\left(\phi + n\frac{\pi}{2}\right)\cos(n\psi), \\ I_{\mathrm{n}} &= v_{\mathrm{out}}(t)\sin(n\omega_{\mathrm{m}}t) \\ &\approx -kJ_{\mathrm{n}}(m)\cos\left(\phi + n\frac{\pi}{2}\right)\sin(n\psi). \end{aligned} \tag{8}$$

k is an amplitude factor common to both I and Q, $J_{\mathrm{n}}(m)$ are Bessel functions. These are sent to the non-linear fit algorithm which uses χ^2 minimisation to determine k, m, ϕ and ψ.

If $f_{\mathrm{m}} = 1\,\mathrm{kHz}$ and $\Delta f = 9\,\mathrm{GHz}$, the arm length difference in the interferometer must be at least $\Delta L = 48\,\mathrm{mm}$ for the self-homodyning to work.

SENSITIVITY

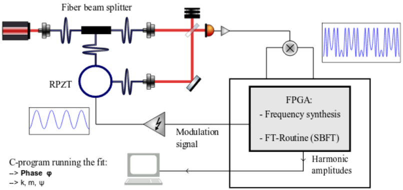
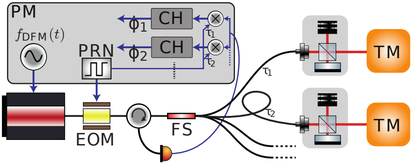


Fig. 8. Schematic of the complete setup for the null measurement with optical signals.

- The LPF phase meter, using deep **phase** modulation, has been shown to have sensitivity of 1 pm $\sqrt{\mathrm{Hz}}^{-1}$ between 1 mHz and 1 Hz
- This is with a matched arm interferometer, with modulation depth $m \approx 9$
- To achieve the same modulation depth here, the arms must be imbalanced to give a round-trip time of 160 ps, or $\Delta L = 48$ mm (for modulation depth of $\Delta f = 9$ GHz and modulation frequency of 1 kHz)

Multiplexing

DFM can in principle be combined with digital interferometry to multiplex multiple optical signals on the same channel (fibre link). Pseudo-random noise is phase-modulated onto input light. Fibre lengths for each link are chosen such that the individual amplitude modulations are not coherent.



Absolute ranging

The modulation depth $m = 2\pi\Delta f\tau$ encodes the light travel time, so can be used for absolute ranging. This works for DPM too. However, either the reference interferometer's delay or the frequency modulation Δf must be measured precisely.

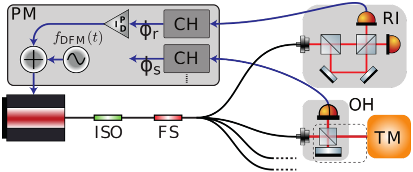
NOISE

The unequal arm lengths required to make DFM work inherently couples laser frequency noise.

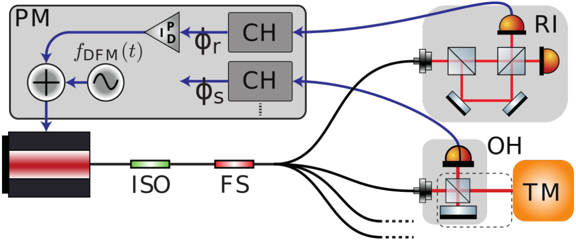
To measure at the level of $1\,\mathrm{pm}\,\sqrt{\mathrm{Hz}}^{-1}$, the frequency noise $\tilde{f}$ must be

$$\tilde{f} < \frac{\Delta L}{L} f = \frac{1\,\mathrm{pm}\,\sqrt{\mathrm{Hz}}^{-1}}{\Delta L/2} \frac{c}{1064\,\mathrm{nm}} \approx 11\,\mathrm{kHz}\,\sqrt{\mathrm{Hz}}^{-1} \tag{9}$$

To stabilise the frequency, a reference interferometer is used with fixed, unequal arm lengths



FREQUENCY NOISE



Excess laser frequency noise is measured as a reference phase ϕ_r by the RI, which can be subtracted from the signal ϕ, or used in a feedback loop (changing the laser's base frequency).

A nice additional feature is that m and ψ_m can also be readout and stabilised with a loop.

OTHER NOISE

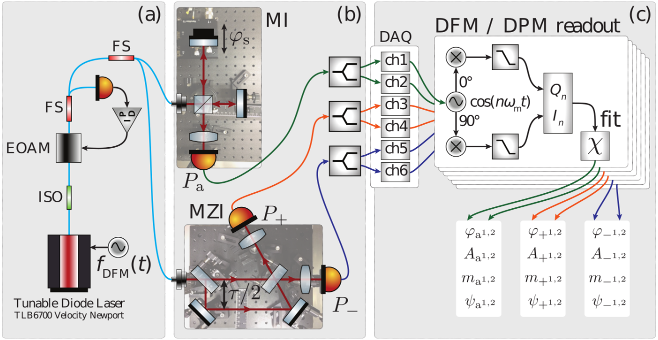
Other noise will influence the interferometers:

- Laser amplitude noise
- Shot noise
- Technical noise (e.g. fibre length noise)
- Non-perfect sinusoidal frequency modulation

These have been a focus of LISA scientists for years, and are not considered great challenges to overcome.

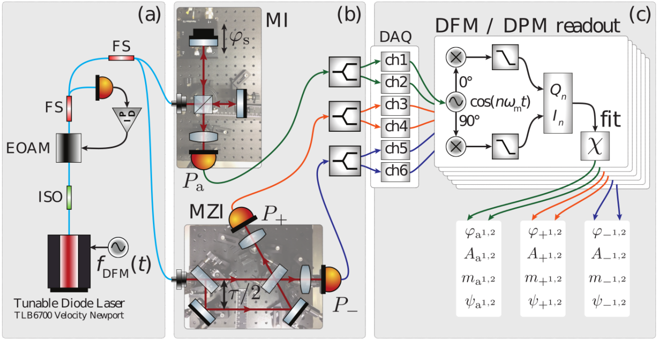
Non-perfect frequency modulation adds beats on critical demodulation frequencies. This is potentially mitigated with a measurement and subtraction device (not considered).

EXPERIMENT



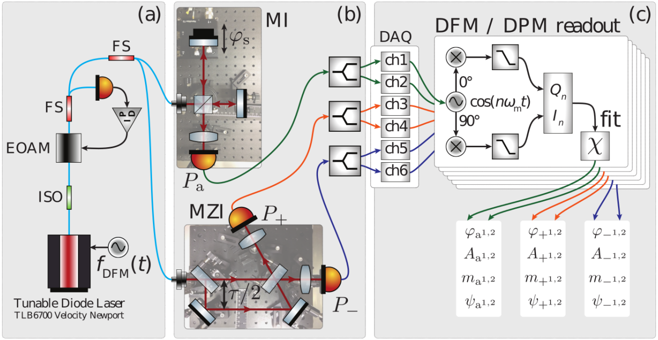
(a)

- Laser source is 1550 nm at 20 mW
- Electro-optic amplitude modulator stabilises the laser's amplitude



(b)

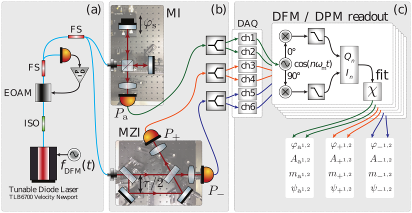
- Light is split into asymmetric Michelson and Mach-Zehnder interferometers
- The MI allows dynamic signals to be tested; the MZ allows two signals with the same magnitude but opposite sign to be extracted



(c)

- ADCs record at 250 kHz
- Post-processing performed with non-linear fitting algorithm on I and Q harmonics

ELECTRONIC NOISE



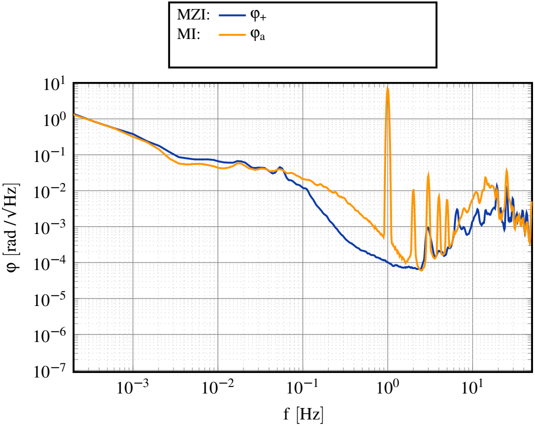
The MZ the signals (ϕ_+ and ϕ_-) can be combined in order to measure different types of noise in various forms. The two MZ signals add up to π:

$$\phi_\pi = \phi_+ + \phi_- = \pi \tag{10}$$

Alternatively, two channels record the same signals, so these can be subtracted:

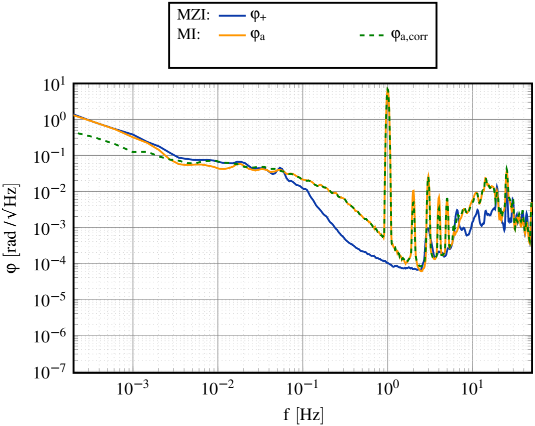
$$\phi_{i,\Delta} = \phi_{i,1} - \phi_{i,2} \approx 0 \tag{11}$$

PHASE MEASUREMENT PERFORMANCE

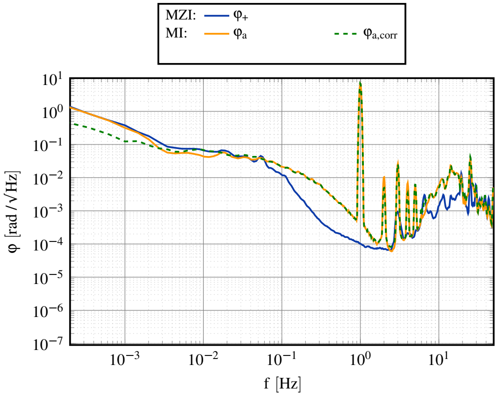


Piezo drive is on MI at 1 Hz. Harmonics clearly present - unsure if from PZT or from non-linearities in readout. Accoustic coupling dominates noise above 1 Hz.

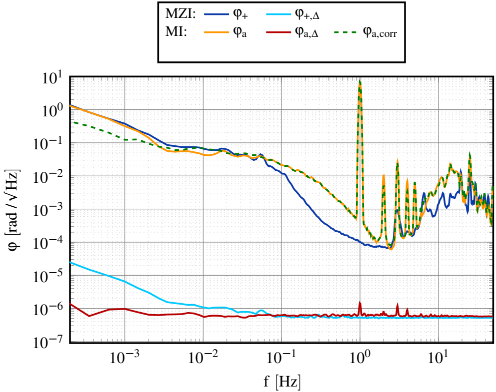
PHASE MEASUREMENT PERFORMANCE



Remember that DFM needs a frequency reference. Using the MZ as a reference for the MI, the sensitivity improves (green trace)

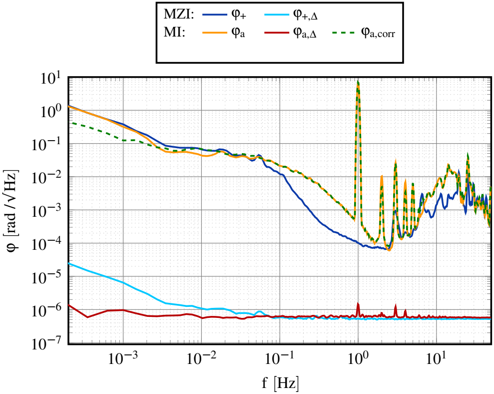


Given that the improvement only happens below 1 mHz, this shows that the limiting noise in the system is thermal fluctuations and air density perturbatations (**not laser noise**). Both effects will be greatly reduced in vacuum.



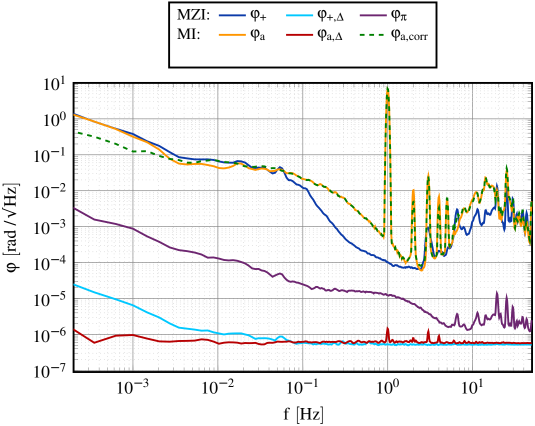
The zero combinations of the MI and MZ signals, by subtracting each pair of channels, shows the electronic noise, likely to be digitisation noise in the ADC. The MI noise contains some of the 1 Hz signal, possibly from non-linearities due to slightly different ADC channel bandwidths.

PHASE MEASUREMENT PERFORMANCE



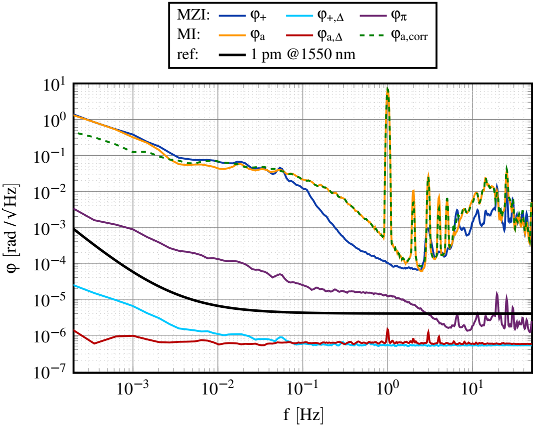
The MZ zero combination, $\phi_{+,\Delta}$, increases at low frequencies, not seen in the MI equivalent. This is possibly due to the larger modulation depth of the MZ, which makes it more sensitive to temperature effects.

PHASE MEASUREMENT PERFORMANCE



The signal-cancelling MZ readout is sensitive to laser amplitude noise **only at the modulation harmonics**, shot noise and ADC noise, giving an idea of the experiment's non-temperature-driven noise

PHASE MEASUREMENT PERFORMANCE



In comparison to the LISA design curve, there is a long way to go from the table top but the results show promise

CONCLUSIONS

CONCLUSIONS

- The LPF optical bench must be shrunken for LISA, so less complex options to classic heterodyne interferometry must be explored
- DFM potentially better than DPM due to ability to share same laser backbone for multiple readouts (like digital interferometry)
- Sensitivity on table top still far off LISA requirement
- Sounds to me like they need to try this in vacuum!