

Beating Heisenberg's Limit

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- 1 How to Build a Gravitational Wave Interferometer
- 2 Beating the Standard Quantum Limit
- 3 Glasgow Sagnac-Speedmeter Experiment



Gravitational Wave Interferometry



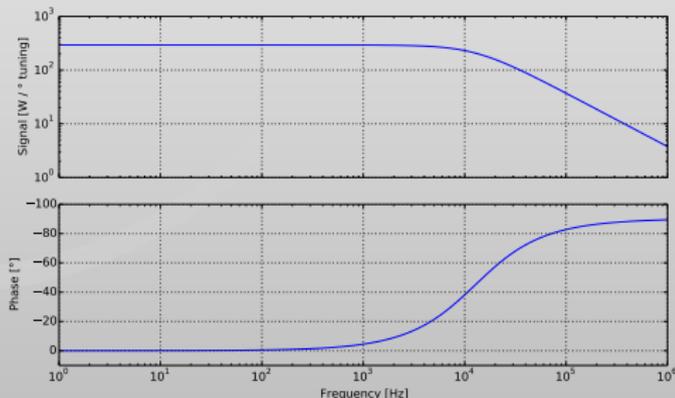
- Build technical infrastructure
 - How long?
 - How much power?
 - How reflective to make the mirrors?
 - Topology?
- Control it
 - Actuation
 - Control systems
- Make it quieter than the thing you want to measure
 - Technical noise sources
 - Fundamental noise sources
- Calibrate its signal in terms of h

Choose an Arm Length

Arm length and mirror reflectivity determine the bandwidth of the detector:

$$f_{\text{pole}} = \frac{\text{FSR}}{\pi} \arcsin \left(\frac{1 - r_1 r_2}{2\sqrt{r_1 r_2}} \right)$$

$$\text{FSR} = \frac{c}{2L}$$



For Advanced LIGO, $f_{\text{pole}} \approx 10$ kHz. NS/NS binary coalescences take place around 40 Hz to 300 Hz.

Use as much power as possible!

More power provides more photons to interact with gravitational waves, and can lower certain types of noise.

With higher powers, can run into stability and heat issues, but in principle these are challenges that can be overcome.

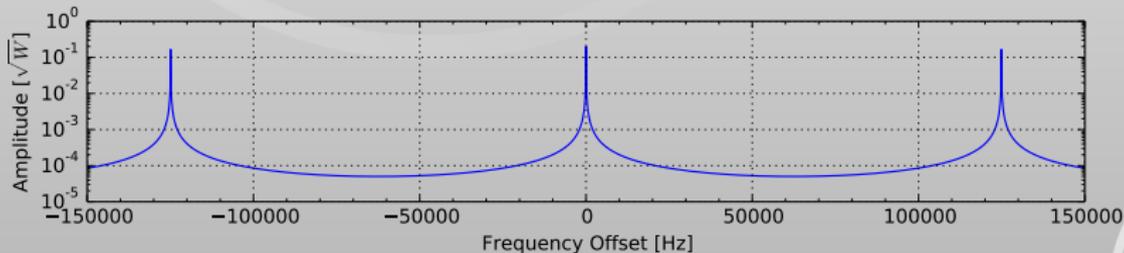
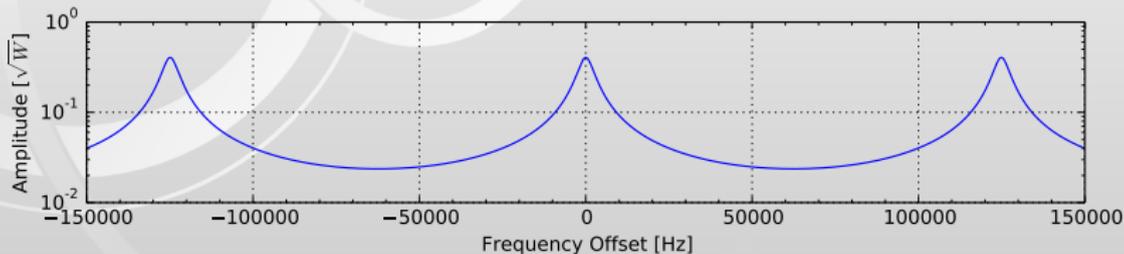
You might think we want as much mirror reflectivity as possible. This is true in theory, but in practice the speed at which the control systems can control the mirrors comes into play.

The cavity mirror reflectivities determine the cavity *finesse*:

$$F \approx \frac{\pi}{1 - r_1 r_2},$$

which is related to how steep the resonance peaks are as cavity mirrors move...

Higher finesse leads to narrower resonances...



...and it's harder to catch the mirror as it swings through.

It turns out that the sensitivity of a Michelson interferometer is determined in the ideal case solely by laser wavelength λ , bandwidth Δf_{BW} and cavity power ϵ .

Mizuno Limit

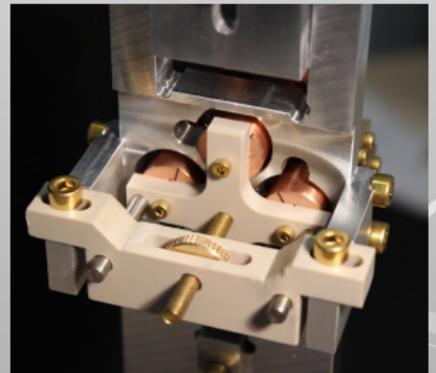
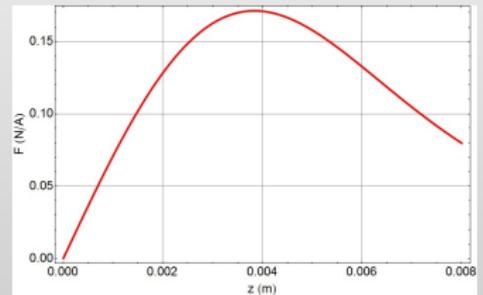
$$\tilde{h} \gtrsim \sqrt{\frac{2\hbar\lambda}{\pi c} \frac{\Delta f_{BW}}{\epsilon}}$$

Jun Mizuno's thesis, Section 3.1.3, Eq. 3 (p56)

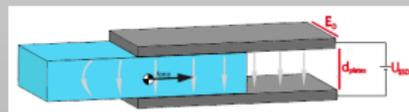
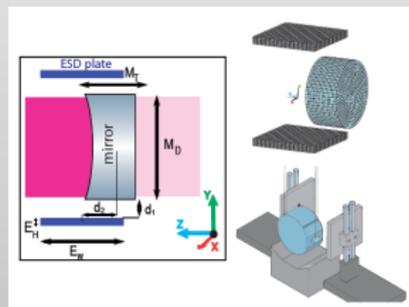
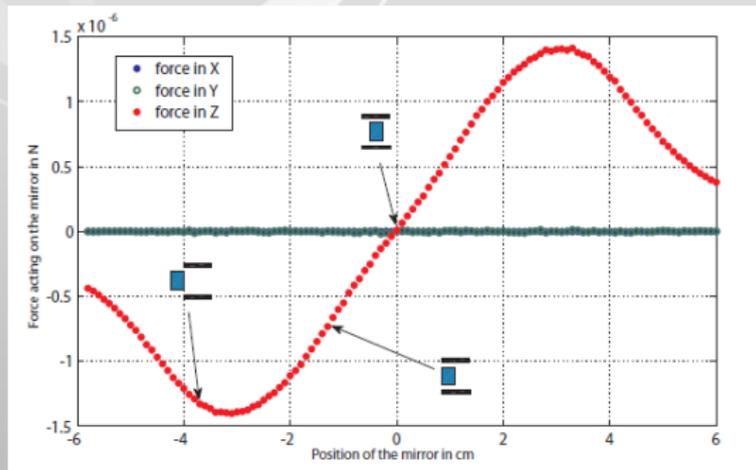
Mirror control is required to get the interferometer into a locked state and keep it there.

This is usually achieved via coil and magnet actuation on the suspensions holding the mirrors.

Actuator noise can be significant. For instance, stray magnetic fields can couple into the actuators and represent a displacement noise source.



For low noise actuation directly on the test mass, electrostatic drives (ESDs) are used. These are low range but low noise actuators.

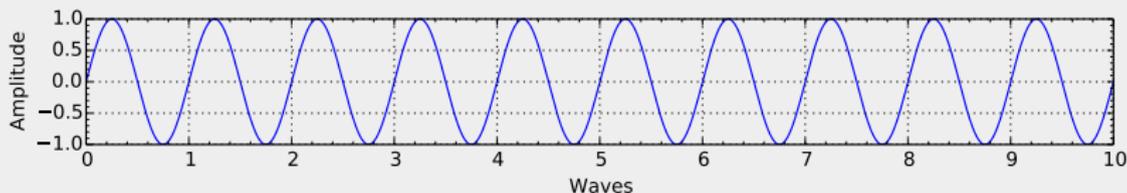


Credit: Wittel et. al. ([arXiv](#))

A GW interferometer needs to be at its **operating point** to be optimally sensitive, with each mirror's position controlled to within as little as 10^{-12} m.

Operating Point

When each cavity within the interferometer is on resonance.



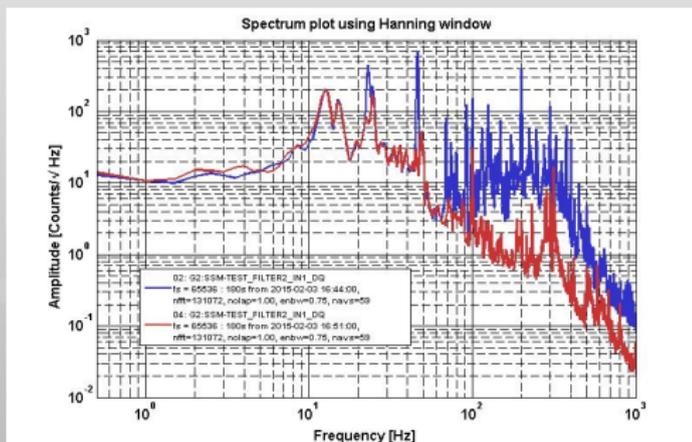
i.e. each cavity fits an integer number of half-wavelengths.

All mirrors are subject to noise from seismic activity on the Earth.

We usually only care about cavity mirror motion, since their motion is enhanced by the cavity.

Suspending optics from pendulums can provide a great deal of isolation, but this is most effective at higher frequencies.

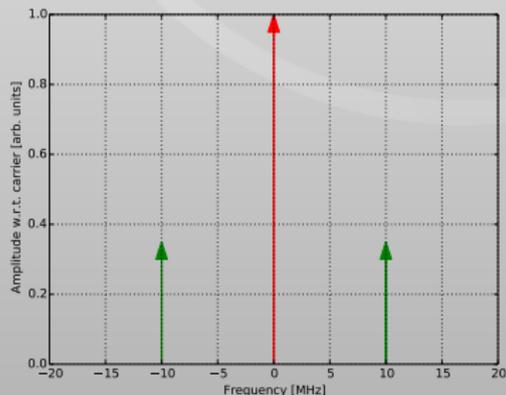
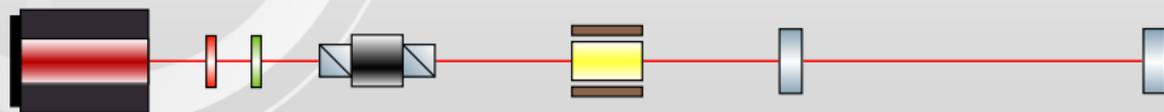
How can we sense cavity mirror motion?



We want the laser (carrier) to be resonant in each cavity.



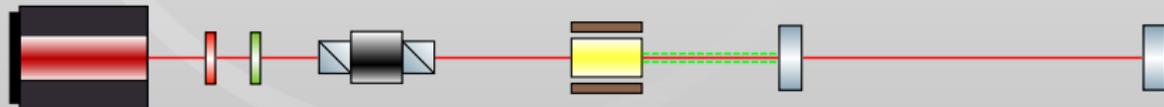
Adding a resonant electro-optic modulator (EOM) lets us superimpose modulation sidebands on the main laser light (carrier).



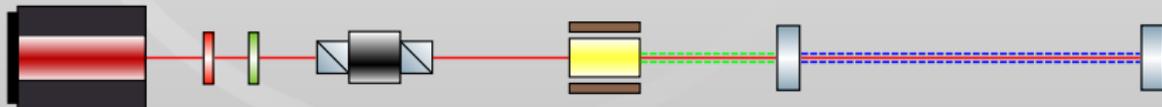
$$E = E_0 e^{i(\omega t + \beta \sin(\omega_m t))}$$

$$\approx E_0 e^{i\omega t} \left[1 + \frac{\beta}{2} e^{i\omega_m t} - \frac{\beta}{2} e^{-i\omega_m t} \right]$$

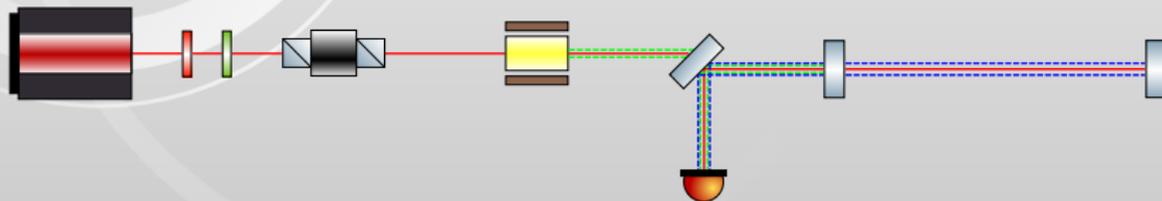
The sidebands are not resonant in the cavity and so are mainly reflected by the first mirror.



Phase changes caused by the movement of the cavity mirrors beat with the reflected sidebands to produce **signal sidebands**.



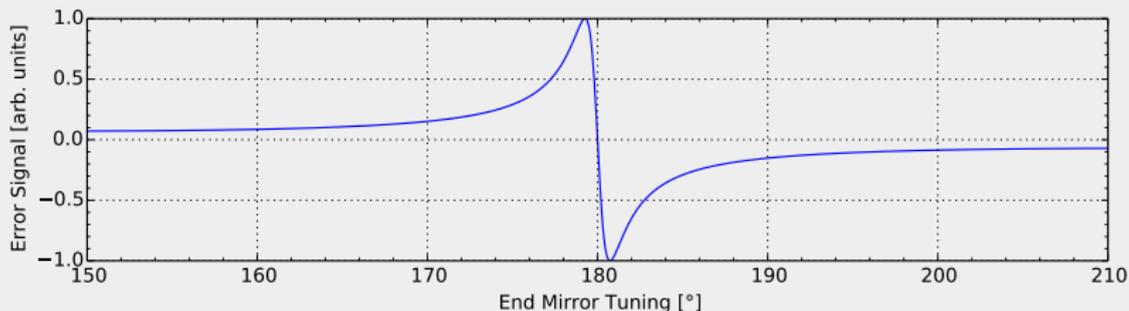
We add a photodiode to view the light reflected by the cavity.



This photodiode is demodulated at the same frequency as the EOM is modulated.

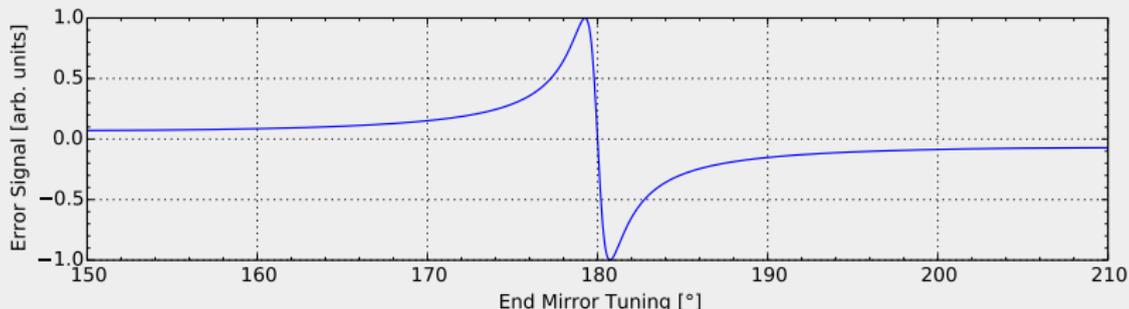
This is called **Pound-Drever-Hall** control.

Pound-Drever-Hall Signal Example



Notice that the signal is **bipolar**. Left of resonance, the cavity gets shorter and the signal is positive. Right of resonance, the cavity is longer and the signal is negative.

Pound-Drever-Hall Signal Example



The slope of the signal at the zero crossing determines the **optical gain** of the interferometer to that particular degree of freedom.

Cavity *finesse*, a function of the mirror reflectivities, determines the slope of the error signal¹.

¹For a simple Fabry-Perot cavity.

For more complicated interferometers, there are many degrees of freedom.

$$\text{DARM} = \frac{L_Y - L_X}{2}$$

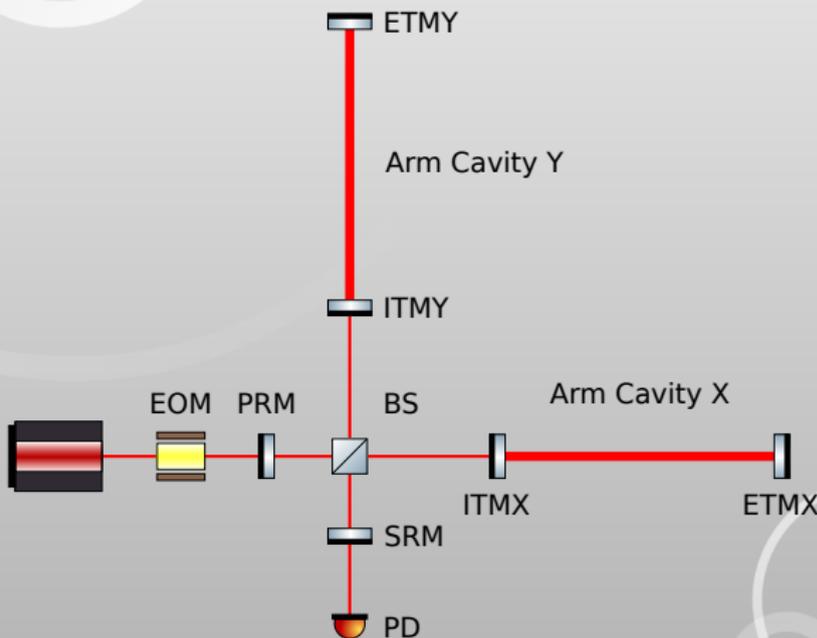
$$\text{CARM} = \frac{L_Y + L_X}{2}$$

$$\text{MICH} = l_Y - l_X$$

$$\text{PRCL} = l_P + \frac{l_Y + l_X}{2}$$

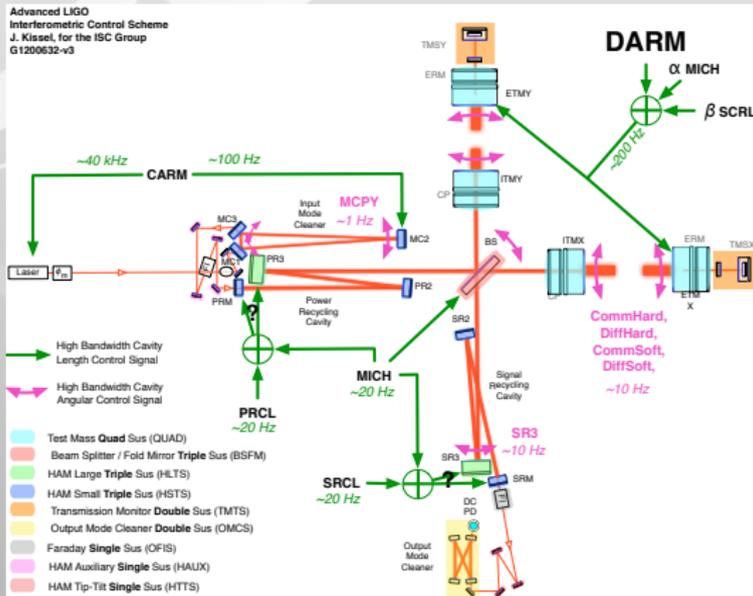
$$\text{SRCL} = l_S + \frac{l_Y + l_X}{2}$$

+ filter cavities,
mode cleaners, etc...





Each degree of freedom is controlled with a control loop. The error signal from each photodiode is fed back to actuators on mirrors.



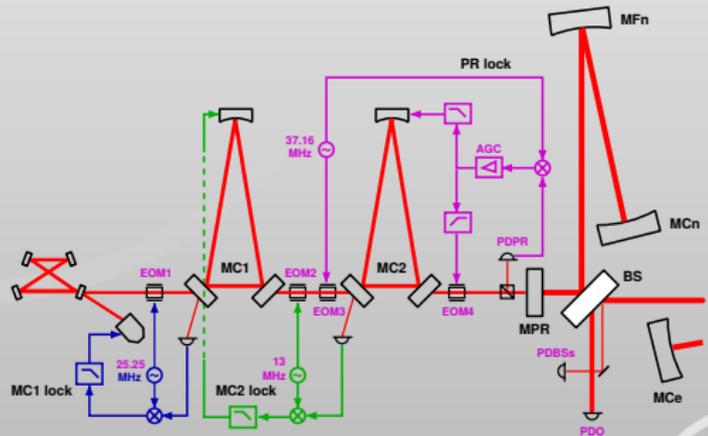
Credit: Jeff Kissel ([LIGO-G1200632](#))



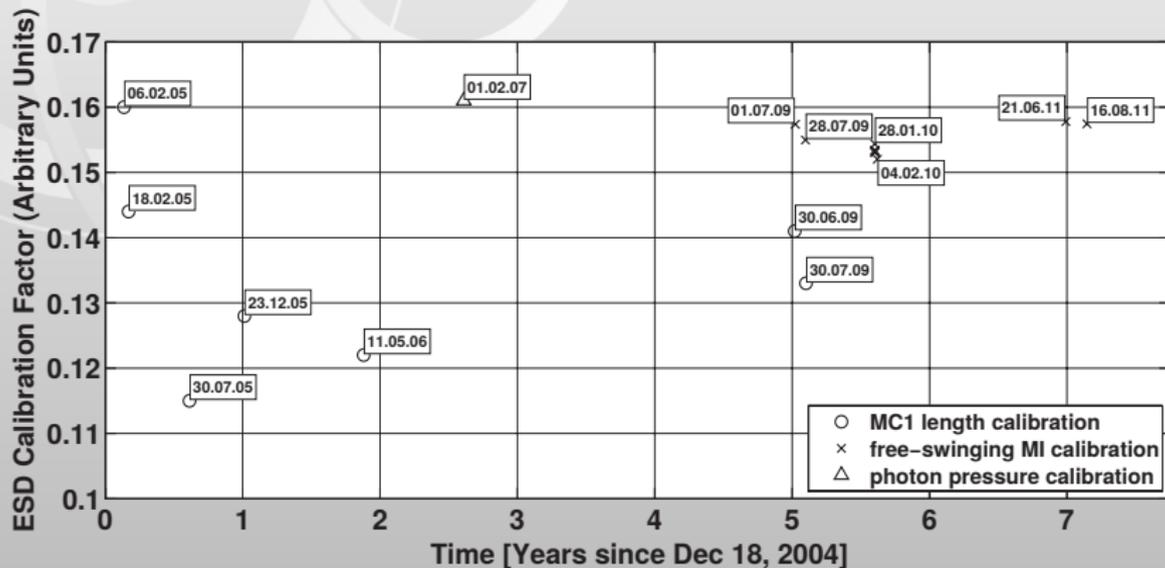
From mirror motion to h

Once you've got a locked interferometer, you need to calibrate its output (volts) in terms of strain (h). This is a whole other topic in itself!

In GEO-HF, for instance, the calibration is done by sequentially calibrating the ESD to the CARM degree of freedom, then the input mode cleaners to the laser stabilisation. The laser's PZT then provides the absolute strain calibration.



Credit: Leong et al. ([DOI: 10.1088/0264-9381/29/6/065001](https://doi.org/10.1088/0264-9381/29/6/065001))



Credit: Leong et al. ([DOI: 10.1088/0264-9381/29/6/065001](https://doi.org/10.1088/0264-9381/29/6/065001))

Test masses need to be quieter than the thing you want to measure. You also need to measure the strain to the required precision without introducing too much additional noise.

That means the laser, mirrors, actuators, control systems and photodetectors in practice set the sensitivity of the interferometer.

These fall into two categories:

- Technical noise sources
- Fundamental noise sources

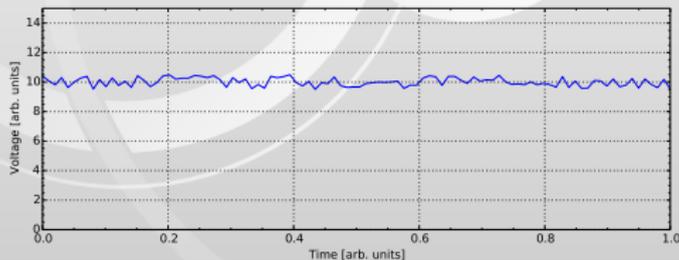
Technical noises represent sources that exist due to the equipment, materials or techniques used, or the temperature at which the interferometer operates.

Thermal noise is a major barrier to sensitivity, both in the test masses and suspensions. This is one of the reasons why LIGO Voyager plans to go cryogenic.

Seismic noise couples ground motion into the test masses at low frequencies where suspensions can perform little damping.

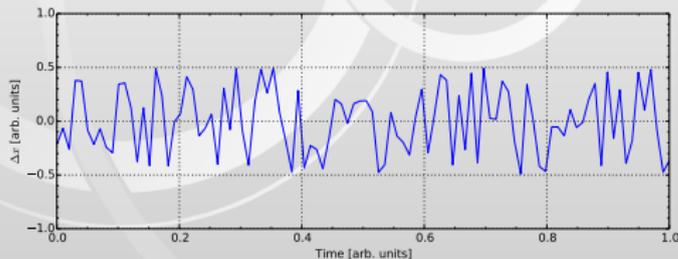
Electronic noise represents noise present in components used to obtain signals from and control the interferometer, such as photodiode dark current, Johnson noise, etc.

Others include laser noise, oscillator noise, gravity gradient noise, etc...

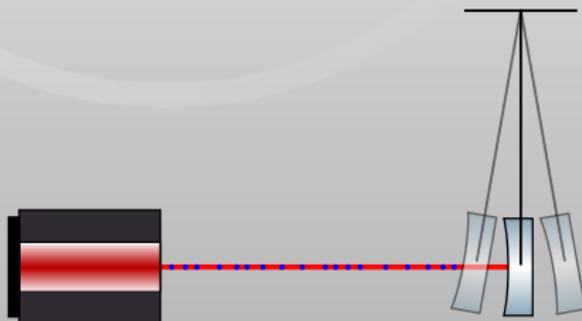


$$h_S(f) = \frac{1}{L} \sqrt{\frac{\hbar c \lambda}{2\pi P}}$$





$$h_{RP}(f) = \frac{1}{mf^2L} \sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$



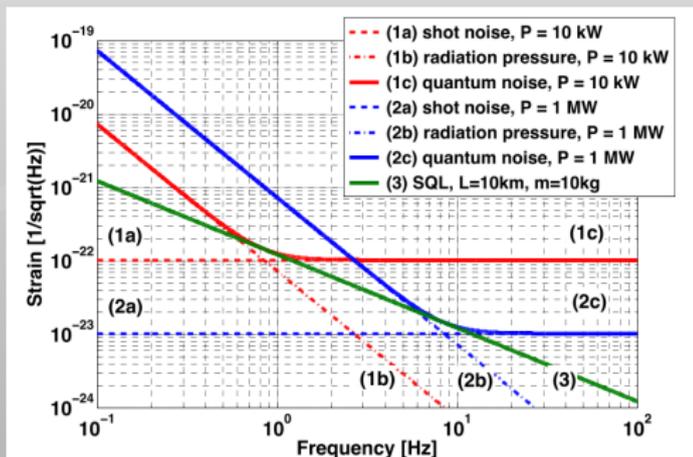
Radiation Pressure Noise

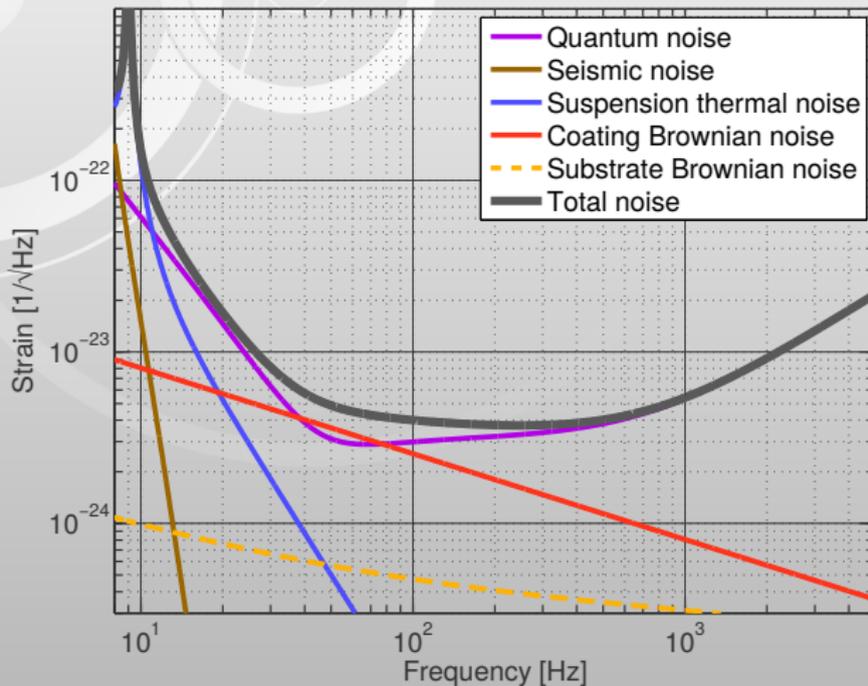
$$h_{RP}(f) = \frac{1}{mf^2L} \sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$

Together, radiation pressure and shot noise combine to place a hard limit on sensitivity at all frequencies for a standard interferometer setup - the SQL. An interferometer only touches the SQL at one frequency but the limit applies to all frequencies.

Shot Noise

$$h_S(f) = \frac{1}{L} \sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

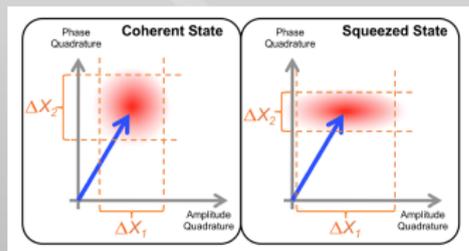




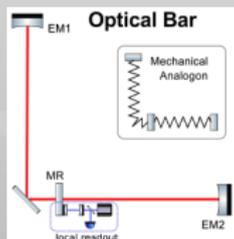
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State of the art detectors are limited by quantum noise nearly over their entire detection range. **How will we improve this performance?**

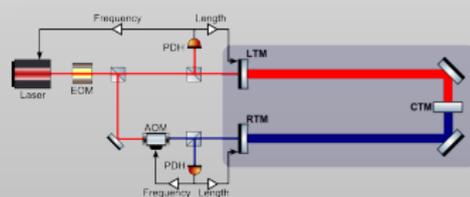
Manipulating the shape of light: **squeezing**



Local readout of an **optical bar**



Optomechanically coupled **optical springs**



Credit: Stefan Hild, [Hollberg workshop](#), Glasgow 2014

So far we used Michelson interferometers to derive strain, by continuously measuring the **displacement** of the mirrors.

Displacement measurements are subject to Heisenberg's Uncertainty Principle. Thus:

$$[\hat{x}(t), \hat{x}(t + \delta t)] \neq 0$$

and

$$[\hat{x}(t), \hat{p}(t)] \neq 0$$

However, in the 1930s, John von Neumann showed that some observables can be measured in pairs continuously without encountering the Heisenberg uncertainty.

One such pair is momentum, which manifests itself as the **speed at which a test mass moves**:

$$[\hat{p}(t), \hat{p}(t + \delta t)] = 0$$



Figure: John von Neumann. By LANL [Public domain], via [Wikimedia Commons](#)

It turns out that a zero-area Sagnac interferometer is automatically a speed meter.

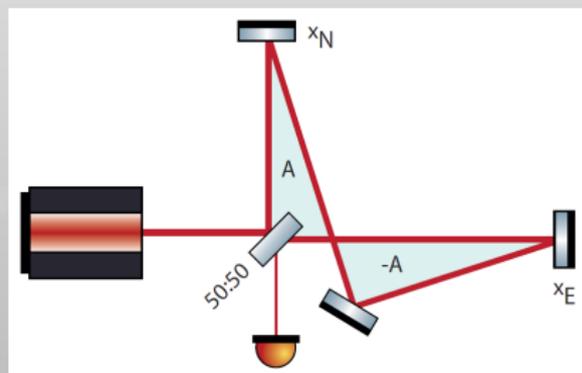
$$\phi_{CW} \propto x_N(t) + x_E(t + \delta t)$$

$$\phi_{CCW} \propto x_E(t) + x_N(t + \delta t)$$

$$\Delta\phi = [x_N(t) - x_N(t + \delta t)] \\ - [x_E(t) - x_E(t + \delta t)]$$

$$\Delta\phi \approx \delta t (\dot{x}_E(t) - \dot{x}_N(t))$$

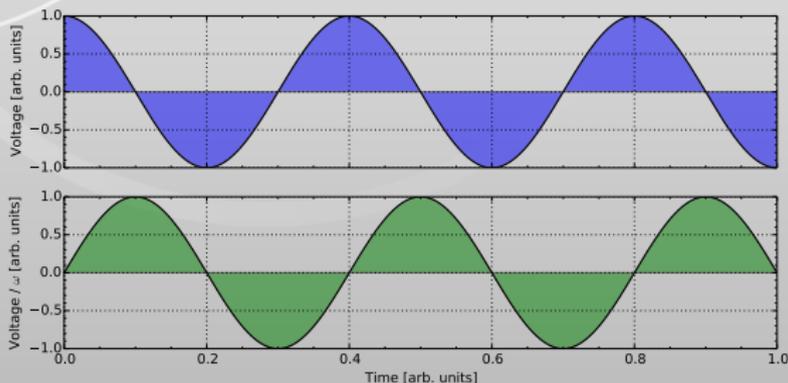
Differential phase is proportional to test mass speed.



Aside: Getting h from speed

There is nothing special required to get strain information from the speed-meter interferometer.

We simply integrate over the speed to get the position, then use existing calibration techniques to get h .



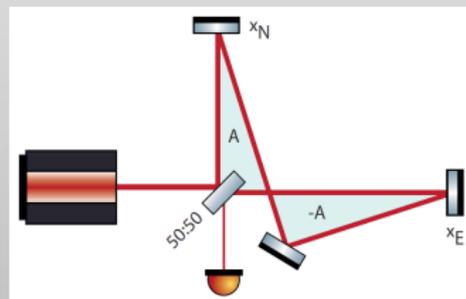
We don't measure the 'DC' position of each mirror, thus we avoid the Uncertainty Principle.

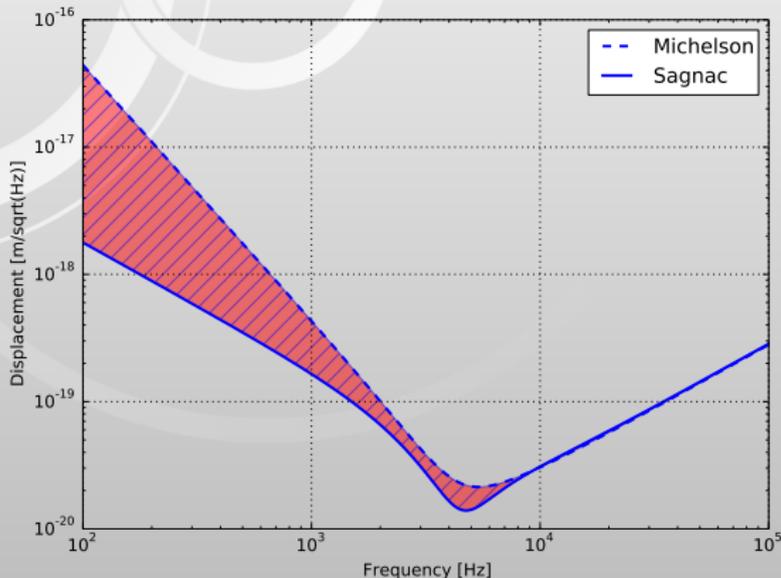
Two counter-propagating beams hit each cavity mirror at a particular time. The two beams impart radiation pressure with a certain phase. To obtain cancellation, the phase difference between these beams has to be 180° .

Can see that:

- cavity power imbalance
- reflectivity imbalance
- losses

will degrade phase difference and thus cancellation of radiation pressure noise.





This is a lossless case. Losses degrade sensitivity and will be important to quantify (more later).

The Speed-Meter configuration has the potential to offer great improvements in sensitivity in future detectors.

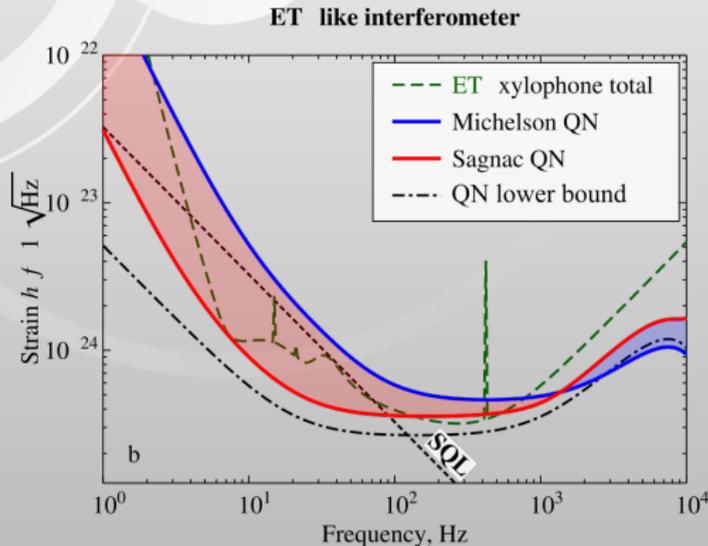


Figure: N. V. Voronchev, S. P. Tarabrin, S. L. Danilishin, [arXiv:1503.01062](https://arxiv.org/abs/1503.01062)

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- Create an ultra-low noise speed-meter testbed which is dominated by radiation pressure noise
- Demonstrate reduced radiation pressure noise over an equivalent Michelson
- Gain experience and understanding of speed-meters for future detector design



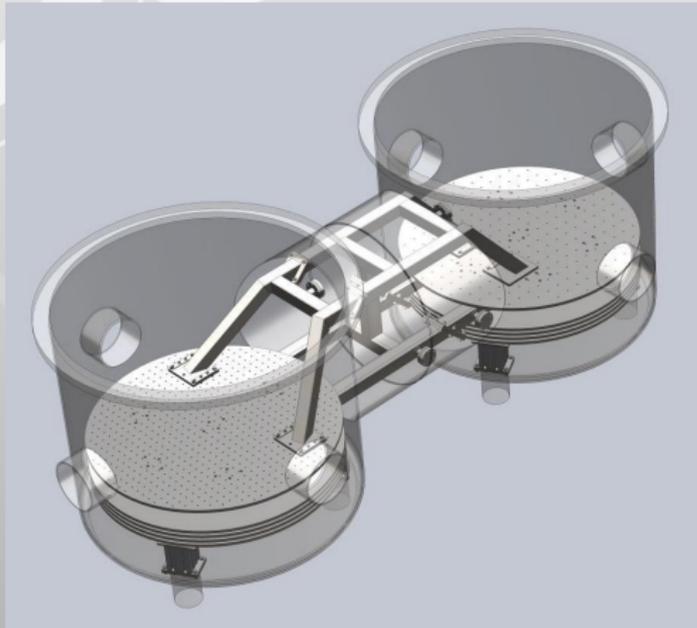
€1.4M ERC grant for
“high risk, high gain”
research

- In vacuum, seismically isolated
- Triangular cavities
- 1 g and 100 g cavity mirrors
- Cavity round-trip length 2.8 m
- In-vacuum, suspended balanced homodyne detector at audio frequencies
- Electrostatic drives for direct actuation on test masses

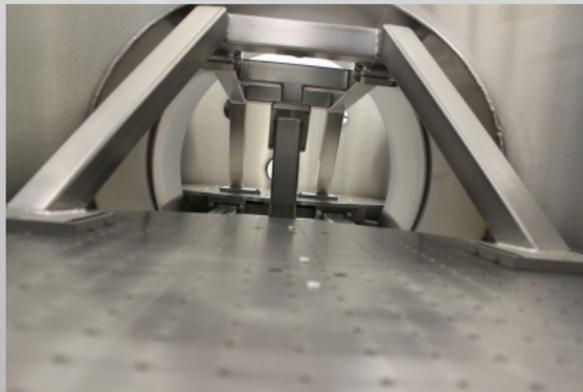




Two 1 m diameter connected tanks. Each tank contains rubber stacks and multiple 30 kg plates for seismic isolation.

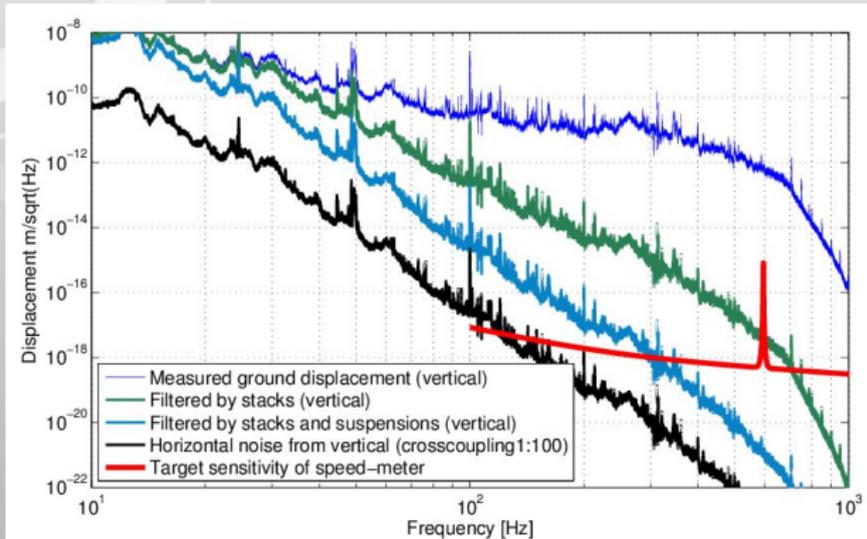


Bridge structure joins the topmost stacks together so their residual seismic motion is common. This makes longitudinal seismic motion common to all mirrors.

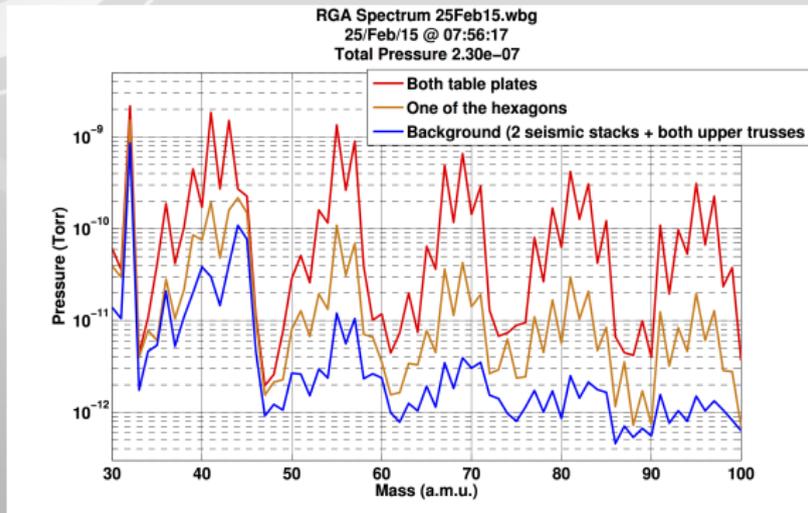


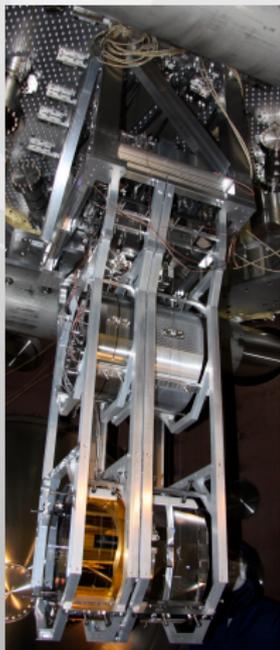


Simulations show that the seismic motion should be damped sufficiently within the 100 Hz to 1 kHz measurement region.



Residual gas analysis shows lots of long-chained hydrocarbons. Probably oil from manufacturing metal structures. Hydrocarbons can settle on mirrors and burn into their surface, leading to loss. Cleaning in progress.





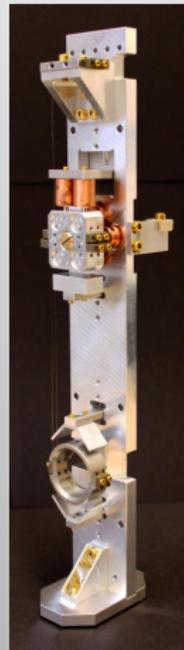
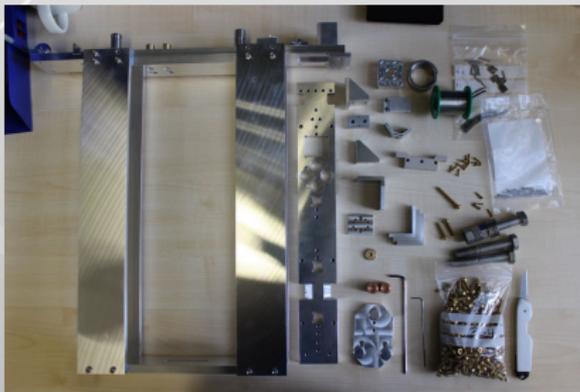
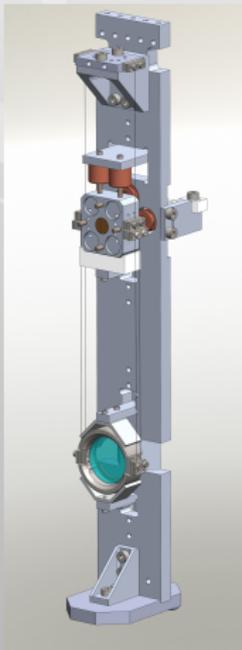
Advanced LIGO: 42 kg



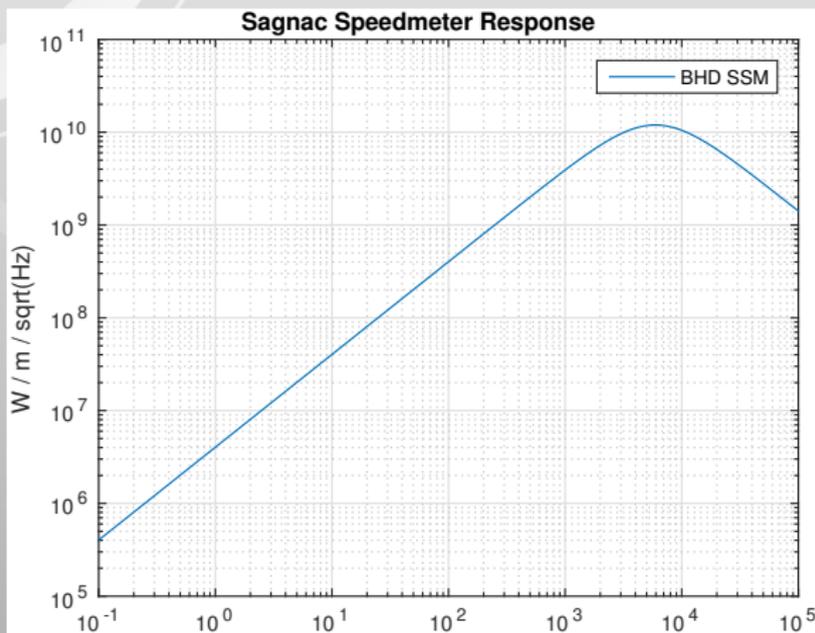
AEI 10 m Prototype:
100 g



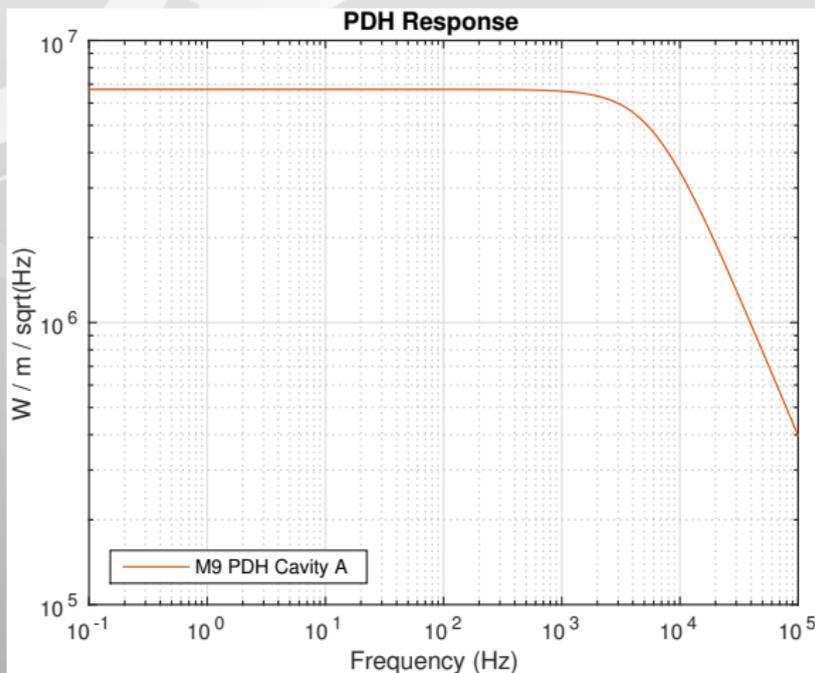
Glasgow Speed-Meter:
1 g



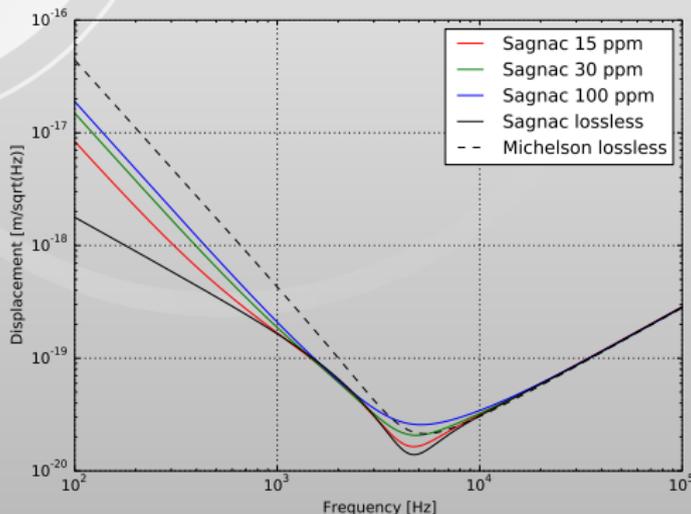
One caveat to the speed-meter is that its response **vanishes at low frequencies**. Below a certain frequency, the interferometer's cavity mirrors are **uncontrollable** due to technical noise in the control system.



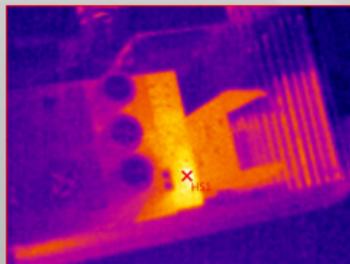
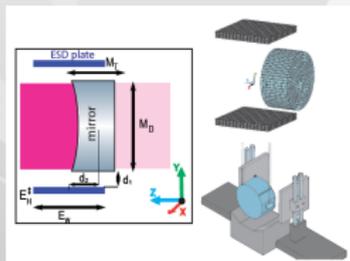
This doesn't happen in a Michelson, where the response is flat (since it's a displacement measurement).



Losses in the Sagnac Speed Meter degrade sensitivity. They reintroduce back-action noise and so we see an extra $\frac{1}{f}$ effect on top of the $\frac{1}{f}$ effect already present.

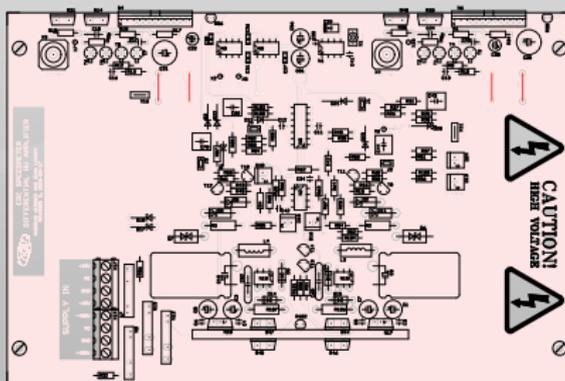


Credit: Danilishin et. al ([arXiv: 1412.0931](https://arxiv.org/abs/1412.0931))



We plan to build the first real-world demonstration of the fringe field ESD design outlined previously.

Simulations suggest we need approximately 800 V actuation range.



- Finish in-vacuum installation of *clean* (!) infrastructure
- Design and build remaining suspensions
- Finalise input optics (mode cleaner, etc.)
- Test ESD concept (parallel experiment, possibly combining BHD test)
- Installation of computerised control and data acquisition system (+ wiring)
- Create sensing and control scheme
- Set final test mass requirements, order optics
- (2016) Start to assemble final experiment and take some data!

<http://speed-meter.eu/>

