Optimal measurement scheme for Advanced LIGO

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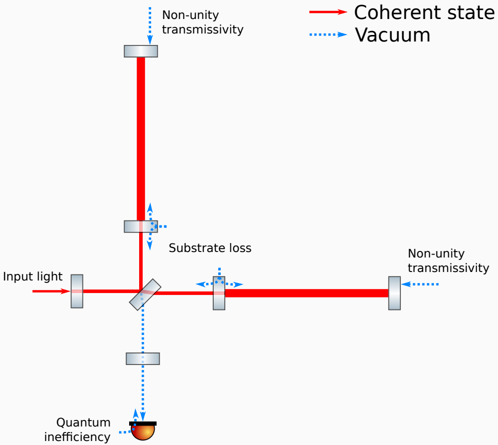
The Paper

[..] Here we discuss the optimal measurement scheme for interferometric gravitational wave detectors and similar setups and find that the previous configured measurement scheme, a single mode intensity measurement, while able to beat the Shot Noise Limit, is outperformed by other measurement schemes in the low power regime, but at high powers, approaches the optimal measurement. [..]

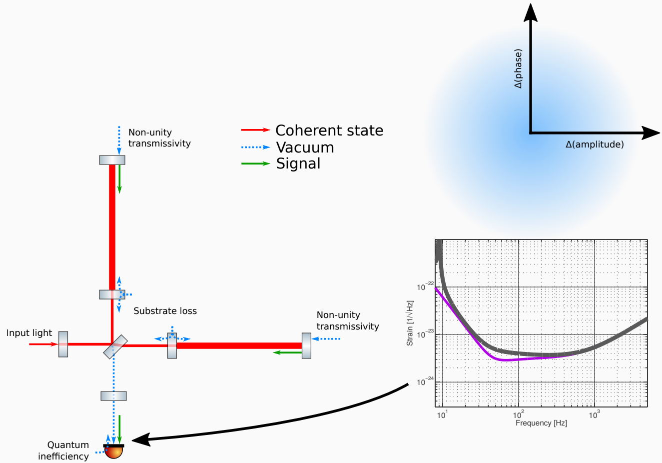
Posted to LIGO P&P (P1600084), not yet peer-reviewed.

Current detector configurations



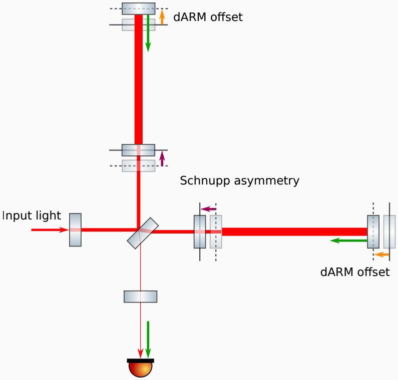
For initial and advanced LIGO and Virgo, the configuration comprised of a coherent state with vacuum entering at the loss points

Current detector configurations



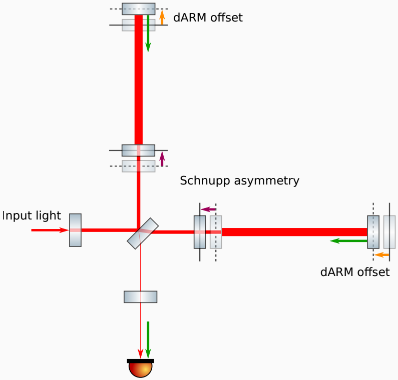
From the ratio of the signal due to mirror motion and the noise from these loss points arises the sensitivity (*noise-to-signal* ratio)

Intensity Readout



- Enhanced and Advanced LIGO and Virgo employ an intensity measurement, achieved by introducing a dARM offset ($\pm 10^{-12}$ m)
- Detuning allows carrier to enter output port via Schnupp asymmetry ($\pm$ 2.52 cm)
- Carrier acts as a local oscillator for signal sidebands, beating to produce an intensity variation at the output
- Works well in terms of noise, since local oscillator is filtered by arm cavities

Intensity Readout

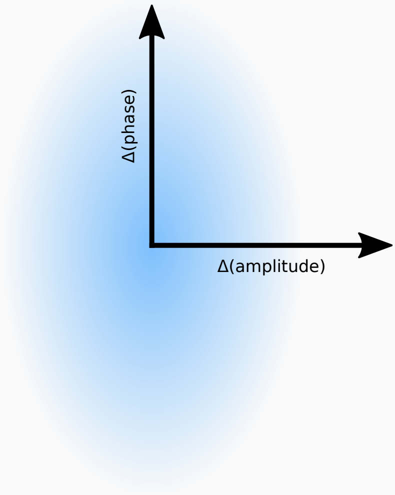


Intensity with DC readout for amplitude $\hat{a}$ is

$$I = \langle \hat{a}^\dagger \hat{a} \rangle$$
$$= \frac{1}{2} \langle \hat{x}^2 + \hat{p}^2 \rangle$$

for amplitude component $\hat{x}$ and phase component $\hat{p}$

Intensity Readout



- This readout is compatible with squeezing
- The amplitude and phase terms, $\hat{x}$ and $\hat{p}$ get larger or smaller
- Since we look at the square term, we can do better than the non-squeezed case
- Squeezing is a planned upgrade for Advanced LIGO, continuing to use intensity readout

Can we do better with other readouts?

Compare, for instance:

- Intensity
- Intensity difference
- Homodyne
- Parity

Other Readouts

Intensity

As discussed, this is a simple measurement of laser power at the output port of the interferometer:

$$I = \langle \hat{a}^\dagger \hat{a} \rangle$$
$$= \frac{1}{2} \langle \hat{x}^2 + \hat{p}^2 \rangle$$

Intensity difference

Involves the measurement of separate intensities, with the results subtracted from one another:

$$I = \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{b}^\dagger \hat{b} \rangle$$

Other Readouts

Homodyne

A phase shifted local oscillator derived from the main beam is overlapped with the main beam (homodyne angle θ):

$$I = \sqrt{2}\left(\cos(\theta)\langle\hat{x}\rangle + \sin(\theta)\langle\hat{p}\rangle\right)$$

Parity

Measurement of whether two photons are entangled (squeezed). If they are, the output is $+1$, and if they aren't, the output is -1.

$$I = (-1)^{\langle\hat{a}^\dagger\hat{a}\rangle}$$

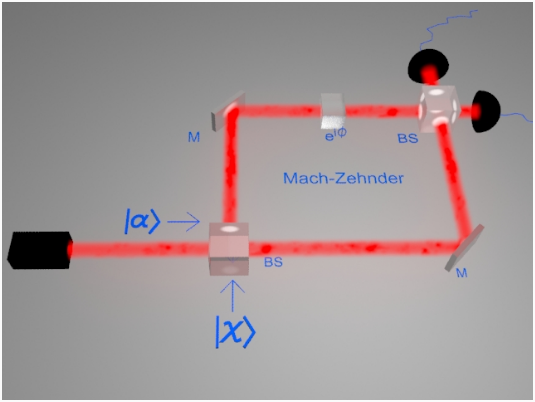
Unsqueezed light has the same expected amplitude and phase variance, so an output's Hermitian conjugate should result in an imaginary part that is equal and opposite, i.e. $(-1)^{-1} = 1$

Can we do better with other readouts?

- Need to calculate not only the readout signal in each case
- But also the physical limit to the sensitivity in each readout
- Each of these readouts leads to a different phase variance
- The smallest variance corresponds to the most sensitive phase measurement

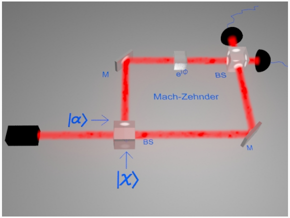
The Model

Authors consider the Mach-Zehnder interferometer, which is mathematically identical to the Michelson, with a phase shift in one arm (a gravitational wave)



Don't have to handle arm cavities, etc. Input light is arm cavity light.

The Model



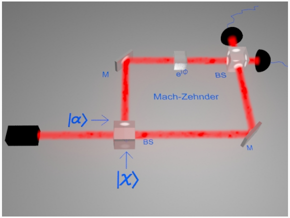
The propagation of light from input to output is modelled with Wigner functions:

$$W(\mathbf{X}) = \frac{1}{\pi^2} e^{\left(-2|\alpha|^2 - p_1^2 + 2\sqrt{2}|\alpha| x_1 - x_1^2 - \left(e^{2r} p_2^2 + e^{-2r} x_2^2\right)\right)}$$

where $\mathbf{X}$ is a vector of amplitude and phase components of each spacial mode:

$$\mathbf{X} = \begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \end{pmatrix}$$

The Model



The beam splitter transformation is described by:

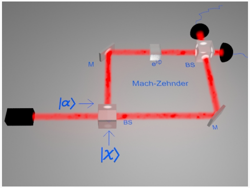
$$\mathrm{BS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

assuming a 50:50 beam splitter

And the phase shift in one arm by:

$$\mathrm{PS}(\phi) = \begin{pmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} & 0 & 0 \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} & 0 & 0 \\ 0 & 0 & \cos\frac{\phi}{2} & \sin\frac{\phi}{2} \\ 0 & 0 & -\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix}$$

The Model



$$W\left(\mathbf{X}\right) = \frac{1}{\pi^2} e^{\left(-2|\alpha|^2 - p_1^2 + 2\sqrt{2}|\alpha|x_1 - x_1^2 - \left(e^{2r}p_2^2 + e^{-2r}x_2^2\right)\right)}$$

The output is then:

$$\begin{pmatrix} x_{1f} \\ p_{1f} \\ x_{2f} \\ p_{2f} \end{pmatrix} = \mathrm{BS} \cdot \mathrm{PS}\left(\phi\right) \cdot \mathrm{BS} \cdot W \begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \end{pmatrix}$$

The measured signal is whatever combination of amplitude and phase the measurement technique provides.

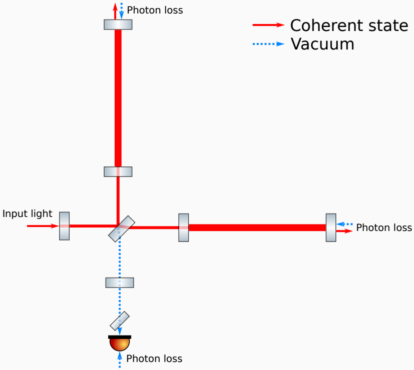
Phase Error

- Can calculate the best phase measurement possible for a given number of photons and level of squeezing
- This is known as the **Quantum Cramér Rao Bound (QCRB)**
- Shows the phase variance for the Mach-Zehnder interferometer to be:

$$\Delta\phi^2_{\mathrm{QCRB}} = \frac{1}{|\alpha|^2 \mathrm{e}^{2r} + \sinh^2(r)}$$

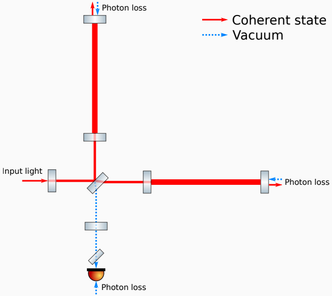
- α represents the square root number of photons, $N_{\mathrm{photon}} = \left|\alpha^2\right|$, and r the squeezing factor
- Aside: $\sinh x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$, so see how the above contains an unsqueezed part and a squeezed part

Modelling Loss



- The QCRB does not contain loss terms
- Assume photon loss and detector inefficiency
- $|\alpha| \to \sqrt{D(1-L)}\,|\alpha|$ for detector efficiency $0 \leq D \leq 1$ and fractional loss $0 \leq L \leq 1$
- $r \to \sinh^{-1}\left(\sqrt{D(1-L)}\sinh r\right)$
- Shows how loss affects squeezing
- Asymmetric loss not considered

Best Sensitivity with Losses



The QCRB then becomes:

$$\Delta\phi^2_{\mathrm{QCRB}} = \frac{1}{\eta\left(|\alpha|^2 \, \mathrm{e}^{2\sinh^{-1}(\eta \sinh r)} + \sinh^2(r)\right)}$$

where $\eta = \sqrt{D(1-L)}$

Best Sensitivity with Losses

- QCRB sets the ultimate limit for any readout, but the effect for individual readout techniques must be calculated
- Calculate the phase variance by taking the output variance and dividing by something resembling the slope of the error signal:

$$\Delta\phi^2 = \frac{\Delta\hat{O}^2}{\left|\frac{\delta\langle\hat{O}\rangle}{\delta\phi}\right|^2}$$

Best Sensitivity with Losses

Analytical results for variances with squeezing:

Parity

$$\Delta\phi^2_{\hat{\Pi}} = \frac{1}{|\alpha|^2 e^{2r} + \sinh^2 r}$$

Intensity difference

$$\Delta\phi^2_{\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}} = \frac{e^{-2r}\left(4|\alpha|^2 + \left(e^{2r}-1\right)^2\right)}{\left(\cosh(2r) - 2|\alpha|^2 - 1\right)^2}$$

Homodyne

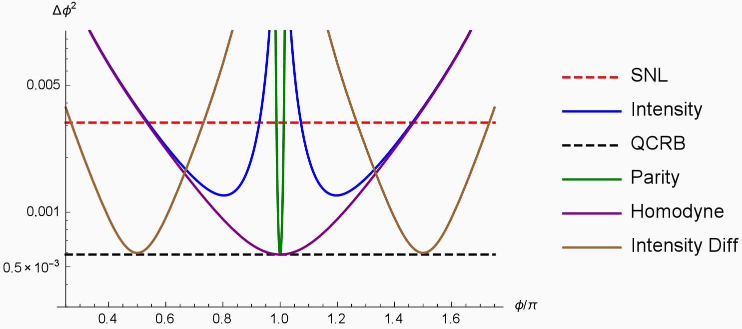
$$\Delta\phi^2_{\hat{x}} = \frac{1}{|\alpha|^2 e^{2r}}$$

Intensity

$$\Delta\phi^2_{\hat{a}^\dagger\hat{a}} = \frac{4|\alpha|^2 e^{-2r} + 2\cosh(2r) + 4\sqrt{2}|\alpha|\sinh(2r) - 2}{\left(\cosh(2r) - 2|\alpha|^2 - 1\right)^2}$$

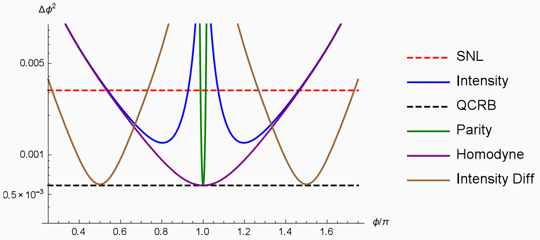
Remember, losses are inserted by substituting $|\alpha| \to \sqrt{D(1-L)}\,|\alpha|$ and $r \to \sinh^{-1}\left(\sqrt{D(1-L)}\sinh r\right)$

Best Sensitivity with Losses



- Loss set to $L = 20\%$ and quantum efficiency $D = 80\%$
- Note that intensity measurements necessarily require dark fringe offset
- Parity performs best, reaching the QCRB; intensity (Advanced LIGO) performs worst
- Homodyne does almost as well as parity (though it is hard to see, it doesn't quite reach QCRB), but over a much wider range

Best Sensitivity with Losses



- To maintain peak sensitivity, the controller needs to keep the output near the trough of the slopes above
- The width of the trough is analogous to the tolerance of the lock, e.g. rms noise
- Clearly homodyne readout is much more forgiving than parity for systems with classical noise
- Parity measurement is furthermore discounted as it requires individual counting of photons, of which in Advanced LIGO there are 10^{24}

Best Sensitivity with Losses

- In fact, at such high powers, **all of the measurements, including intensity measurements**, asymptote to the QCRB level, so there is no significant sensitivity advantage to any technique
- However, the intensity measurement scheme contains a term linearly proportional to α:

Intensity

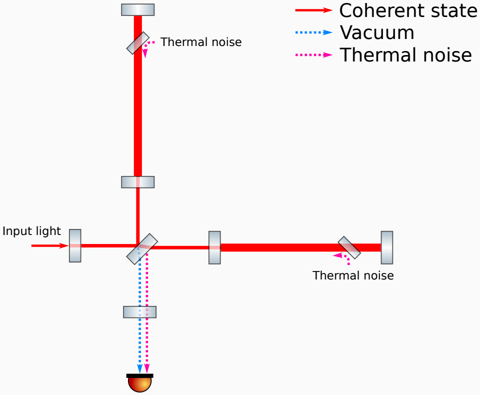
$$\Delta\phi^2_{\hat{a}^\dagger\hat{a}} = \frac{4|\alpha|^2 e^{-2r} + 2\cosh(2r) + 4\sqrt{2}|\alpha|\sinh(2r) - 2}{\left(\cosh(2r) - 2|\alpha|^2 - 1\right)^2}$$

- This means the optimal sensitivity changes if the source changes, e.g. laser noise or squeezer drift
- The other readouts have near-constant optimal readout phase ($\phi_{\hat{\Pi}} \to \pi$, $\phi_{\hat{x}} \to \pi$, $\phi_{\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}} \to \frac{\pi}{2}$)
- **Homodyne readout appears to be the most realistic upgrade for Advanced LIGO**

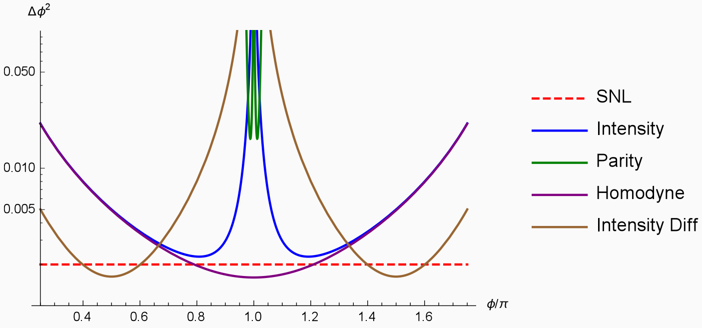
Questions?

Thermal Noise

- So far, have only considered photon loss and quantum inefficiency
- What about thermal noise?
- Introducing beam splitters to the arms allows a tunable amount of noise to be created by exciting photons into a thermal state



Thermal Noise



- Room temperature operation with μm wavelength is immune to this noise (we're not talking about coatings), so this only applies to microwaves ($\lambda > 1\,\mathrm{mm}$)
- Parity significantly above the shot noise limit
- Intensity readout no longer reaches shot noise
- Homodyne's advantage over intensity readout is reduced, but still beats shot noise

ET-LF

ET-LF will still have vastly too much power to benefit significantly from parity measurement

$$|\alpha|^2 = \frac{P}{\hbar\omega_0}$$

Low power regime is $|\alpha|^2 < 500$, i.e. $P \approx 10^{-18}\,\mathrm{W}$.