Evaluating mirror (alignment) systems using the optical sensing matrix

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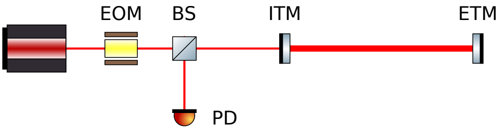
based on a paper by
Maddalena Mantovani and **Andreas Freise**

Operating Point

A GW interferometer needs to be at its operating point to be optimally sensitive, with each mirror's position controlled to within as little as 10^{-12} m.

Simple interferometers are (usually) simple to control

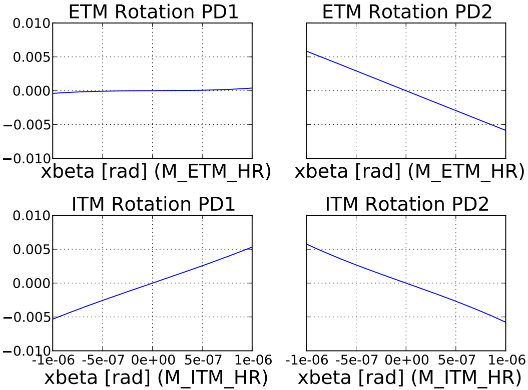
- Two-mirror Fabry-Perot cavities
- Simple Michelsons



There are some complicating factors - more of these later.

Probes

You need to sense the movement of the mirrors inside your interferometer. We do this with ‘probes’ (typically photodiodes). These might see a DC signal of some sort, or a demodulated RF signal via the Pound-Drever-Hall technique.



The above image shows the slopes of the Pound-Drever-Hall error signals during mirror rotation for an example Fabry-Perot cavity.

Optical Matrix

The behaviour of an interferometer on its operating point* can be expressed in the form of the *optical matrix*:

$$\boxed{\mathbf{M} = m_{i,j}}$$

for i probes and j degrees of freedom (e.g. tilt, rotation, longitudinal motion of each mirror).

We can then map mirror motion for a particular degree of freedom B_j to photodiode signals s_i via the relation:

$$\boxed{m_{i,j} \cdot B_j = s_i}$$

*Getting an interferometer to its operating point is someone else's problem. For the purposes of control we assume the interferometer is near to its locked state.

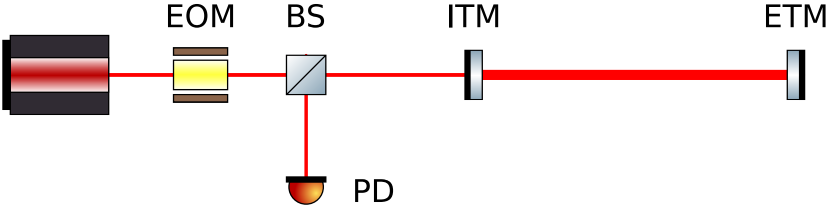
Optical Matrix

$$\boxed{m_{i,j} \cdot B_j = s_i}$$

For a simple Fabry-Perot with two mirrors and one photodiode, $m_{i,j}$ might look something like this:

$$\begin{array}{c} \\ \text{PD} \end{array} \begin{array}{c} \begin{array}{cc} \text{ITM} & \text{ETM} \end{array} \\ \begin{pmatrix} 0.5 & -0.5 \end{pmatrix}, \end{array}$$

i.e. the PD contains a mixture of the movement signals from both the ITM and ETM. **Note**: values are actually complex but shown as real here for clarity.



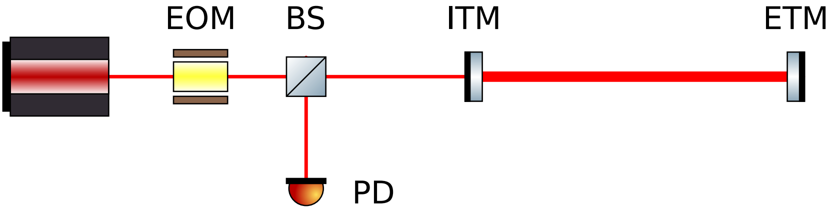
Optical Matrix: Aside

$$\boxed{m_{i,j} \cdot B_j = s_i}$$

Another way of expressing this matrix is with *degrees of freedom* representing the different forms of relative motion the arm cavity mirrors can have. We can define common and differential motion, for example:

$$\begin{array}{c} \\ \text{PD} \end{array} \begin{array}{c} \begin{array}{cc} \text{Common} & \text{Differential} \end{array} \\ \begin{pmatrix} 0 & \quad\quad 1 \end{pmatrix} \end{array}.$$

The PD sees a linear combination of the motion of the ITM and ETM.



Optical Matrix

Units are straightforward to work out:

- The photodiode measures in watts
- Mirror motion is either metres or radians (or anything else you might wish to call it)
- **Hence** the interferometer matrix **M** is in watts per metre or watts per radian

$$\boxed{m_{i,j}[\mathbf{W}/\mathbf{m}] \cdot B_j[\mathbf{m}] = s_i[\mathbf{W}]}$$

Optical Matrix

Inverting **M** would give us units of [**m**/**W**] (or [**rad**/**W**]).

We can just look at our photodiodes, multiply the signal by the inverse of our interferometer matrix and we end up with a set of mirror motions!

$$\boxed{\mathbf{M}^{-1}\mathbf{S} = \mathbf{M}^{-1}\mathbf{M} \cdot \mathbf{B} = \mathbf{B}}$$

Assuming we can feed this motion information back to the mirrors in a control loop, we can keep the interferometer locked.

Optical Matrix

This places a constraint on our control matrix: it must be invertible:

$$\boxed{\det(\mathbf{M}) \neq 0}$$

Physically, this means that the matrix must map the full mirror longitudinal / angular space. Even if we don't care about a particular mirror's motion to detect GWs (e.g. the power recycling mirror), we still need to know about it to be able to control the arm cavities (to detect GWs).

Optical Matrix

An example of **M** for an uncontrollable interferometer:

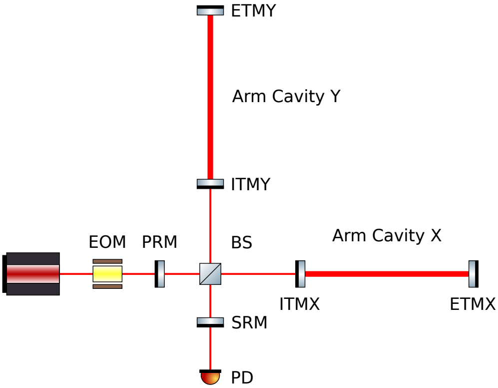
$$\begin{array}{c} \\ \text{PD 1} \\ \text{PD 2} \end{array} \begin{array}{c} \begin{array}{cc} \text{ITM Pitch} & \text{ETM Pitch} \end{array} \\ \left(\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right) \end{array}$$

This matrix's determinant is 0. It shows the effect of pitch of both the ITM and ETM is degenerate; you can't separate the different mirror motions between the two photodiodes even with elementary row operations.

Advanced Interferometers

Gravitational wave detectors typically have more mirrors than just the three involved in a Michelson. That means there are some additional dynamics to complicate things:

- Second arm cavity
- Power recycling mirror
- Signal recycling mirror
- (Not shown) Schnupp asymmetry
- (Not shown) ITM detuning for DC readout
- (Not shown) Signal recycling detuning



Advanced Interferometers

A realistic control matrix for an advanced interferometer might look something like this:

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array}
\begin{array}{c}
\begin{array}{cccccccccc} \text{XH} & \text{XS} & \text{YH} & \text{YS} & \text{SRCH} & \text{SRCS} & \text{PRCH} & \text{PRCS} & \text{MCHH} & \text{MCHS} \end{array} \\
\begin{pmatrix}
1 & x & x & x & x & x & x & x & x & x \\
x & 1 & x & x & x & x & x & x & x & x \\
x & x & 1 & x & x & x & x & x & x & x \\
x & x & x & 1 & x & x & x & x & x & x \\
x & x & x & x & 1 & x & x & x & x & x \\
x & x & x & x & x & 1 & x & x & x & x \\
x & x & x & x & x & x & 1 & x & x & x \\
x & x & x & x & x & x & x & 1 & x & x \\
x & x & x & x & x & x & x & x & 1 & x \\
x & x & x & x & x & x & x & x & x & 1
\end{pmatrix}
\end{array}
$$

Advanced Interferometers

More complications, not part of the 2nd generation detectors (at least, the core parts):

- High finesse cavities
- Coupled mirrors (Neil's experiment, Khalili cavities, etc.)

It is difficult to calculate the interferometer matrix **M** analytically, which is why we usually simulate things with FINESSE or Optickle.

Advanced Interferometers

We need a way of evaluating whether a control matrix for a given set of probes is good or not!

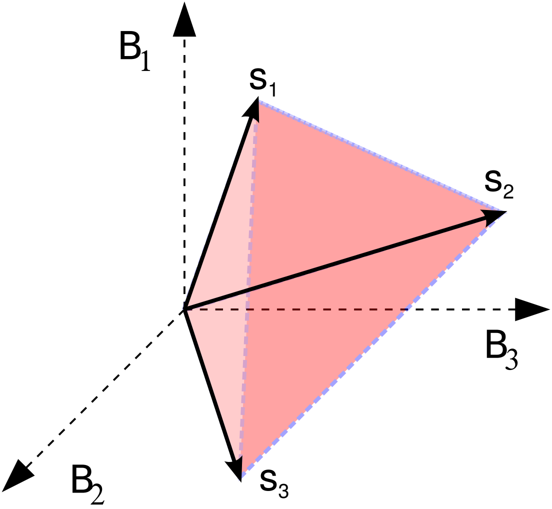
The paper describes a way of evaluating control matrices.

Evaluating Control Matrices

The determinant does not necessarily tell us anything about the controllability of a matrix. We can instead use the *wedge product*:

$$V = |\det(\mathbf{M})|$$

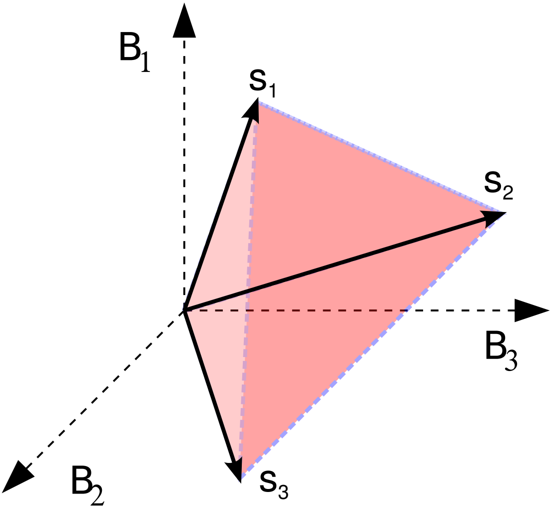
This gives us a volume V which is the volume spanned by the normalised vectors s_i in the mirror space $\mathbf{B}$.



Evaluating Control Matrices

This volume represents the controllability of the system:

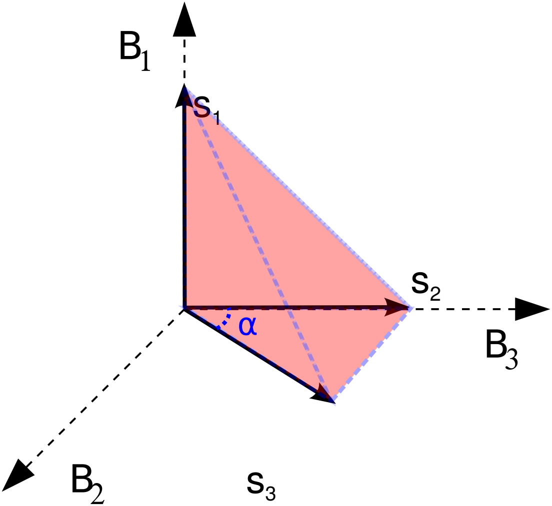
- A zero volume (i.e. all sensors sensing the same DOFs) means the system is uncontrollable
- A volume of $V = 1$ would be perfectly decoupled in each DOF
- $0 < V < 1$ is **not intuitive**



Evaluating Control Matrices

For $0 < V < 1$ we can instead use a special technique:

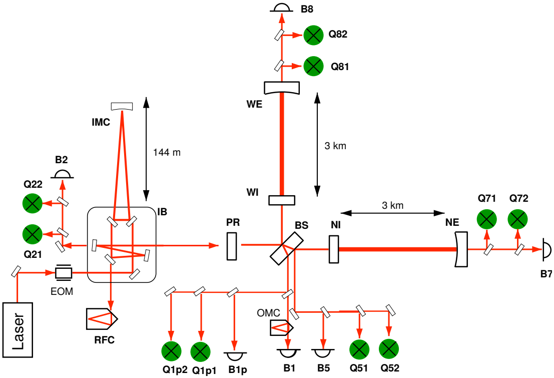
- Make all vectors orthogonal except one
- This last vector can then be offset by the angle α making the volume equal to V
- It turns out that $\alpha = \arcsin\left(\left|\det\left(\mathbf{M}\right)\right|\right)$
- α is then a figure of merit for the controllability of the interferometer given a control matrix



Application to Virgo

Virgo contains more sensors than degrees of freedom:

- Control matrix not square
- No defined determinant



The controllability can therefore only be calculated by working out the best subset of sensors to control the interferometer.

Application to Virgo

In Virgo, this quality parameter α was used to determine the original angular control scheme's suitability. It was found to have $\alpha \approx 2^\circ$ (bad). Simulations showed that introducing an additional sideband frequency to the interferometer made $\alpha \approx 40^\circ$!

So, **it works**!

Aside: Non-stationary systems

In reality, we have noise to deal with as well as the optical response of an interferometer to our probes. The operating point might change with time, so control matrices which are not only optimal but also robust to small changes are favourable.

It turns out we can visualise uncertainties with cones on the mirror DOF space.